Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations | 1. Thesis and Dissertation Collection, all items

1950

A study of the effect of span and number of bays on economy of framing material in single story gabled structures utilizing articulated wedge-beam framing.

Spangler, William Stark

Rensselaer Polytechnic Institute

http://hdl.handle.net/10945/14647

Downloaded from NPS Archive: Calhoun

DUDLEY KNOX LIBRARY

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

http://www.nps.edu/library

A STUDY OF THE EFFECT OF SPAN AND NUMBER OF BAYS ON ECONOMY OF FRAMING MATERIAL IN SINGLE STORY GABLED STRUCTURES UTILIZING ARTICULATED WEDGE-BEAM FRAMING

WILLIAM STARK SPANGLER

Thesis
S664

Library
U. S. Naval Postgraduate School
Annapolis, Md.

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

A STUDY OF THE EFFECT OF SPAN AND NUMBER OF BAYS ON ECONOMY OF FRAMING MATERIAL IN SINGLE STORY GABLED STRUCTURES UTILIZING ARTICULATED WEDGE-BEAM FRAMING

William Stark Spangler

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Civil Engineering

Rensselaer Polytechnic Institute
Troy, New York
May 1950

Thesis
S6664

CONTENTS

PART I

ACKNOWLEDGMENTS iv

PART II

INTRODUCTION 1

PART III

GENERAL DISCUSSION 4
3-1. The Problem 4
3-2. Procedure Followed 4
Discussion of Procedure 6
3-3. Design Data and Assumptions 7
Frame Dimensions and Construction . . . 7
Frame Loading 9
Allowable Stresses 9
Members 9
3-4. Method of Analysis 10
3-4a. Single Bay Structures 10
3-4b. Multiple Bay Structures 10
Deflection Equations 13
Illustrative Problem 15
Development of Simplified Method . . . 17
Illustrative Problem Using Simplified Method 19

12848

3-5. Member Design 20

3-6. Results 22

3-7. Discussion of Curves 23

3-8. Analysis of Curves and Conclusions 24

3-9. Suggestions for Further Study 26

PART IV

COMPUTATIONS 27

4-1. Single Bay Structures 29

Detailed Design 29

4-2. Two Bay Structures 32

Detailed Design 32

4-3. Three Bay Structures 35

Detailed Design 35

4-4. Four Bay Structures 38

Detailed Design 38

4-5. Tabulation of Weights 42

4-6. Some Properties of Selected Wedge-Beam Sections 43

PART V

GRAPHS AND FIGURES 44

5-1. Graphs 44

1. Frame Weight vs Total Span 44

2. Frame Weight vs Number of Bays . . 45

5-2. Figures 46

1. Construction and Properties of a Wedge-Beam 46

2. Frame Utilizing Wedge-Beams 47

3. Dimensions of Designed Frames . . . 48

4. Cutting Structures for Analysis . . 49

5. Structure of Illustrative Problem . 50

PART VI

BIBLIOGRAPHY 51

APPENDIX

Tables and Forms Developed by A. Amirikian

I. ACKNOWLEDGMENTS

After reading the article, "Future Developments in Welded Steel Buildings," by Mr. Arsham Amirikian, Head Designing Engineer with the Bureau of Yards and Docks of the U. S. Navy, I decided that I particularly desired to do a thesis in connection with "Articulated Wedge-Beam Framing" as developed by Mr. Amirikian. Mr. Amirikian offered suggestions and valuable advice both in selecting my subject and in carrying this work to its conclusion. For his interest and guidance I am very grateful, and I hope that my work will prove of some value in his further development of wedge-beam framing. My work on this thesis has done much to broaden my outlook on the whole field of structural design.

I wish to acknowledge with appreciation the assistance of Admiral Lewis B. Combs, Head of the Department of Civil Engineering at Rensselaer Polytechnic Institute, in making it possible for me to do this particular thesis and in obtaining some of the literature used. I also extend thanks to Professor J. Sterling Kinney, Professor of Structural Engineering at Rensselaer Polytechnic Institute, for his many helpful suggestions as faculty advisor throughout the course of my work.

WILLIAM S. SPANGLER

Troy, New York
May 1950

II. INTRODUCTION

The objective of this work is to determine the most economical system of bays and spans for the frame of a structure utilized to cover a given width of flooring. The structure is to be designed according to the concepts of articulated wedge-beam framing as they are adapted to a one story gabled steel frame of the shape ordinarily used in mill buildings. The specific object of the study is to obtain a series of curves showing the weight of structural steel per square foot of floor area covered plotted against the number of equal length bays used. Probably the best way to illustrate how such curves could be used is by an example. Given the problem to design a warehouse 150 feet wide using the principles of articulated wedge-beam framing. The curves would indicate which of the following structural arrangements would probably be most economical from a material standpoint: (a) a single 150 foot span, (b) two seventy-five foot spans, (c) three fifty foot spans, or indirectly (d) two forty and two thirty-five foot spans.

The concept of articulated wedge-beam framing has been developed by Mr. Arsham Amirikian, Head Designing Engineer with the Bureau of Yards and Docks, U. S. Navy. Mr. Amirikian has extensively covered the aspects of this type of framing and is now in the process of publishing the complete theory including analytical formulae and tables for use in the construction industry. A brief outline of the basic principles

involved in wedge-beam framing is contained in a preliminary article.[1] Conventional design methods do not fully utilize the materials in flexural members in that they require the choice of an entire member based on some maximum stress occuring at only one point in the member. This exaggerated inefficiency in the use of materials is often partially corrected by placing a haunch at the section of high stress. Wedge-beam framing much more completely corrects this fault by the use of tapered beams (See Fig. 1), which are designed to furnish a varying section modulus to conform roughly to the moment diagram of the member being designed, thus avoiding large portions of understressed material.

The concept of an articulated frame composed of wedge-beam members is based on the action of a simple cantilever beam in flexure. The cantilever beam has the unique property of having a consistent pattern of bending since the moment diagram inevitably varies from a minimum at the free end to a maximum at the fixed end, no matter what system of lateral loading is applied. It is seen from Fig. 1 that the section modulus of a wedge-beam varies in the same general way as the moment diagram of the simple cantilever beam. Cantilever action is obtained in a structure by the use of the articulated phase of the theory. Consider a single bay gabled frame (See Fig. 2). If a pin is introduced at the crown and if the knee joints are rigid, the crown will be sufficiently free to cause each

---

[1] Amirikian, A. "Future Developments in Welded Steel Buildings." The Welding Journal, XVII (Aug.1948) 593-599.

roof member to act as a cantilever beam fixed at the knee. Further, the columns are joined to the foundation by a pin connection. Referring to Fig. 2 it is seen that the moment diagram of the structure is analogous to a series of cantilever beam moment diagrams with fixed ends at the knees of the frame. By selecting the proper taper for the members the section modulus diagram furnished can follow closely the section modulus diagram required. As a sidelight it is noted that the gabled structure shown in Fig. 2 is statically determinate while its counterpart in an ordinary rigid frame is indeterminate to at least one degree.

This explanation of the underlying principles of wedge-beam framing is admittedly brief and is presented in the nature of background material. Only one of the possible applications of wedge-beam framing has been mentioned. Also any discussion of the expected savings in cost and weight resulting from the application of this principle instead of the principles of ordinary rigid framing or truss framing is considered outside the scope of this study. Economic studies could be made as a continuation of this thesis. The principles of articulated wedge-beam framing as conceived by Mr. Amirikian are still new and contain many possibilities for further investigations. In fact, many investigations will have to be made to firmly establish the practicability of the theory.

III. GENERAL DISCUSSION

3-1. The Problem.

The specific problem undertaken in this thesis was to obtain a series of curves which would show what arrangement of bays and spans would give the greatest economy of structural framing material for a single story gabled steel frame designed to cover a given total span. The structure was constructed according to Mr. Amirikian's concept of articulated wedge-beam framing. It was decided that the desired results could best be shown by a group of curves of weight of structural framing material per square foot of floor area covered plotted against the number of bays used. The curve group would contain individual curves for various total spans with the total span divided into the number ofequal span bays indicated on the graph abcissa. Thus the problem became one of designing a number of one story gabled steel frames utilizing articulated wedge-beam framing. Enough frames using different numbers of bays for a given total span had to be designed in order to obtain a good curve of weight versus number of bays for that total span. Further, results for a sufficient number and spread of total spans were needed to indicate whether or not a generalized statement concerning the problem could be made from the pattern of the series of curves.

3-2. Procedure Followed.

It was felt that the mechanics of designing the frames would be facilitated if the designs were pointed towards an

initial series of curves of weight of framing material in pounds per square foot of floor area plotted against total span in feet. For this series separate curves were obtained for single bay, two bay, three bay, and four bay structures. From these initial curves the final curves of weight of material versus number of bays were obtained. By using this procedure it was possible to utilize somewhat repeating groups of spans per bay for each of the initial curves. This use of the same spans per bay for each of the various number of bays greatly facilitated checking the design computations.

With this general procedure determined, an outline of the order of attack was as follows:

1. Determination of the basic design data and assumptions including the frame dimensions and loading and the various factors governing allowable stresses.

2. Selection of spans for the single bay frames and design of those frames. Frames were designed for spans of 40, 60, 80, 120, and 150 feet. These spans were, of course, both the span per bay and the total span.

3. Design of frames for the two bay structure. Frames for the spans listed were designed.

Span per bay	Total span
20 feet	40 feet
40	80
60	120
80	160
120	240
150	300

4. Design of frames for the three bay structure. Frames for the spans listed were designed.

Span per bay	Total span
20 feet	60 feet
40	120
60	180
80	240
100	300

5. Design of the frames for the four bay structure. Frames for the spans listed were designed.

Span per bay	Total span
20 feet	80 feet
40	160
60	240
80	320
100	400

6. Plot of initial series of curves of Frame Weight in pounds per square foot of floor area versus Total Span in feet for single bay frames, two equal bays, three equal bays and four equal bays. See Graph 1.

7. Plot of the final series of curves using points from the initial curves. These curves show Frame Weight in pounds per square foot of floor area versus Number of Equal Bays for total spans of 60, 80, 120, 160, and 240 feet. See Graph 2.

8. Analysis of curves.

Discussion of Procedure. A total of twenty-two frames were designed and a point for each of these frames was plotted on the initial series of curves (Graph 1). In plotting the final series of curves (Graph 2) only eighteen points were used, and of these only thirteen were points actually plotted on Graph 1. This does not indicate an inaccuracy in Graph 2 as a glance at Graph 1 shows that interpolation between points was quite easy and would not introduce any appreciable error. However, it does indicate a possible falicy in the general procedure used. Since time very definitely limited the extent of the work on this thesis, it is possible that points for a five bay structure could have been obtained if the design work had been pointed directly towards the final series of curves. If the curves could have been extended to five bays, the value of the curves would have been increased materially.

The method of analysis used for the multiple bay frames is the method described by Mr. Amirikian in his current articles. It is discussed in Article 3-4.

In designing the frame as a whole and the individual members of the frame only skeleton designs were done. That is to say that while the Steel Construction Manual of the A.I.S.C. was followed in designing the members, no design or weight allowance was made for items such as the welding or stiffeners required. Since the end result of each design was a weight, it was felt that the elimination of these items introduced a negligible error in that weight particularly as it was in terms of pounds per square foot of floor area.

No design was prepared for any part of the structure except the actual bay framing itself. It was assumed that the other parts of the structures, besides being of secondary interest, would have an approximately equal weight expressed in pounds per square foot of floor area. Thus the only variable entering into the graph ordinates was the weight of the framing itself. Accordingly the ordinates plotted were only the weight of the frame reduced to pounds per square foot. Plotting only the weight of bay framing material tended to emphasize the effect of increasing the number of bays for a given span, and was therefore felt to be in keeping with the object of the thesis.

3-3. Design Data and Assumptions.

Frame Dimensions and Construction. See Fig. 3. The following assumed data concerning the frames pertains to all the structures designed except as specifically noted.

Column height from base of column to intersection of working lines of column and roof members is 20 feet.

Bay depth is 20 feet.

Span per bay is the distance between adjacent column working lines. Total span is the distance between working lines of the two exterior columns.

Gable rise above top of columns for:

spans per bay up to and including 20 feet is 5 feet,

spans above 20 feet up to and including 60 feet is 10 feet,

spans above 60 feet up to and including 120 feet is 15 feet,

spans above 120 feet is 20 feet. These groupings were used to facilitate the use of Mr. Amirikian's design tables and still provide a somewhat uniform yet practical rise to span ratio.

Lateral support of members (perpendicular to Y-Y axis) is as follows.

Exterior columns have lateral support throughout.

Roof members have lateral support throughout.

Interior columns are laterally supported by bracing at a point 15 feet above the floor level. These assumptions were made as a compromise between simplicity and practicality in the design. They tended to simplify, and so speed up, the design work without injuring the results of the project. The 15 foot unsupported length of the interior columns was arrived at through consideration of the problem's statement. It was assumed that the area covered by the structure must be as clear as possible of structural elements, and that a wall running

along the interior columns would be objectionable.

Frame Loading. See Fig. 3. The following applies to all frames. The entire structure is loaded with a uniform gravity load on the roof of 50 pounds per square foot of horizontal area covered. With a 20 foot bay depth this gave a uniform gravity load of one kip per foot of span on the bay framing. No horizontal loading of any type was assumed. The assumed loading pattern was not in accordance with general building code specifications. The 50 pounds per square foot was adequate allowance for a combined dead and uniform snow live load for most areas. No attempt was made to deal with wind loads because the method of handling such loads varies with different designers. Further, it was not felt that the addition of a wind load would have any material effect on the relative shapes and positions of the final curves. A similar argument held with regard to the vertical loads and their application. Simplicity of design was the governing factor in the choice of frame loading.

Allowable Stresses. The A.I.S.C. Steel Construction Manual was used throughout the design of members, It has been previously mentioned that stiffeners were not designed, but it was assumed that they would be used where necessary to make any designed members acceptable.

Members. See Fig. 1. It was recommended and proved profitable to use the minimum cut (minimum value of "e" in Fig. 1) that could be used in practice. Accordingly, e was taken as two inches wherever possible. This allows just enough

clearance between flanges at the small end of the wedge-beam to conveniently weld the two sections together. This is further discussed in Article 3-5.

3-4. Method of Analysis.

This discussion of the method of analysis is divided into two general sections. The first section deals with the single bay structures and the second with the multiple bay structures.

3-4a. Single Bay Structure.

A single bay, one story gable frame utilizing articulated wedge-beam framing (Fig. 2) is a statically determinate structure. It is made statically determinate through the articulated phase of the design which has a pin at the crown joint and pin foundation connections. Thus there are four reaction components to be found, a horizontal and a vertical component at each of the foundation pins. Because of the pin at the crown, about which the moment must be zero, four condition equations may be written for the structure. These are the usual equations of statics concerning the sum of the external horizontal and vertical forces and moments plus an equation stating that the moment at the crown pin is equal to zero. These simultaneous equations may be solved directly to find the reaction components. Having found the reaction components, the moments required to complete the analysis may be written directly.

3-4b. Multiple Bay Structures.

The multiple bay, one story gable frames utilizing articulated wedge-beam framing designed in this work were

analyzed by a method developed by Mr. Amirikian. This discussion deals only briefly with the method as it applies to one story gable frames loaded with a uniform gravity loading. Mr. Amirikian is now publishing articles which develope and illustrate the method completely in its application to different types of articulated wedge-beam frames under varying conditions of loading.

The use of articulated wedge-beam framing as opposed to ordinary rigid framing reduces the number of unknowns which must be determined by methods of indeterminate structures. If the number of bays contained in a frame is indicated by "n", then the number of indeterminate unknowns, "x", may be expressed by the following relations:

For unsymmetrical loading -

Even number of bays, $x = n-1$
Odd number of bays, $x = n-1$

For symmetrical loading and framing -

Even number of bays, $x = n/2$
Odd number of bays, $x = (n-1)/2$

A frame of the type designed in this work may be considered to consist of a series of simple frames joined together by crown joint pins. See Fig. 4. For the prupose of analysis the division or cutting of a frame into its component simple frames does not require special rules. In most cases the separation is made at the crown joint in every other bay of an assembly. Thus in Fig. 4, the three bay frame is cut at the crown hinge of the second bay to form simple frames 1 and 2. The four bay frame may be cut as shown into simple frames 1, 2

and 3. The unknowns in any frame consist of the two hinge reactions at each joint linking adjacent simple frames. The number of unknowns for a symmetrical three bay frame that is symmetrically loaded (Fig. 4) is reduced to one in accordance with the equations given above by the fact of its symmetry. The vertical force at hinge B must be zero. Similarly the analysis of the four bay frame could by simplified to take advantage of symmetry by cutting the frame at hinges B and C. The cuts and forces shown in Fig. 4 illustrate the general case.

In order to determine the values of these hinge reactions their interrelationship is first established. This is obtained from the consideration that two adjacent simple frames will have the same deflection at a common link. The next step is to solve the resulting series of simultaneous equations. More specifically, the procedure to be followed in the analysis consists of the following:

1. Subdivide the main assembly or framing into its component simple frames, and indicate the unknown hinge forces at each cut joint. This will result in a series of statically determinate simple frames, each subjected to unknown H and V forces on each side in addition to its own loading.

2. Draw the moment diagram of each simple frame due to each unknown H and V force as well as one for the given loading of the frame. This results in a series of moment diagrams.

3. Using the moment diagrams in (2), write an expression for the horizontal and vertical deflection at each cut joint

of each simple frame.

4. Equate the deflections of adjacent simple frames. This will result in a set of simultaneous equations, the number of which will correspond to the number of the unknown hinge reactions.

5. Using the equations in (4) solve for the unknown hinge reactions.

Deflection Equations. In general, two equations can be derived from the two deflections at each connecting joint of adjacent simple frames. As stated above, these are obtained by equating the horizontal and vertical deflections of the joint from one frame to the corresponding expressions in the other frame. The deflection of a simple frame is given by the usual expression:

$$\Delta_D = \int \frac{M_s m_s ds}{E I_x} \qquad (1)$$

in which Δ_D is the deflection of a joint D, measured in the desired direction, M_s is the simple frame moment due to the applied loading, m_s is the simple frame moment due to a unit load applied at the joint in the desired direction of the deflection, ds is an element of length, E and I are the modulus of elasticity and moment of inertia, respectively; and the summation includes all the members of the frame.

Since each member of the framing has a rigid joint at one end and a hinged connection at the other, the bending moment diagram of each member is similar to that of a cantilever beam. The moment may be due to an end force, such as

the hinge reaction normal to the axis, or to a given loading applied between the two ends of a member. Now, let P_M represent the end force or normal hinge reaction, which produces the simple moment M_s in a member, and p_m the end force which produces a moment m_s . Then the contribution of a member to the deflection of the simple frame, Eq. (1), may be considered as the deflection, Δ, of a cantilever subjected to a load of $P_M \times p_m$ at its end. It is given by the relation:

$$\Delta = P_M \ p_m \cdot \frac{l^3}{EI} \cdot k \qquad (2)$$

in which k is a constant depending on the shape of the member, l is the length and I is the moment of inertia of the small end of a tapered or wedge-beam member.

The deflection of a member due to a loading applied between the ends is the same as that of a simple cantilever. For a concentrated load, P, the expression is:

$$\Delta_p = \frac{P\,l^3}{EI} \cdot k_p \qquad (2a)$$

in which k_p is a constant depending on the shape of the member and the position of the load.

Similarly, for a uniformly distributed loading the expression is:

$$\Delta_w = \frac{w\,l^4}{EI} \cdot k_w \qquad (2b)$$

k_w being the shape and the load constant for this case.

With this concept, to obtain the deflection Δ of a simple frame, it is necessary to sum up the deflections of its component members as given by Eqs. (2). The factors

k, k_p and k_w for various types of structural members have been evaluated and tabulated by Mr. Amirikian. The tables giving k values for the type of members used in this work are in the Appendix.

__Illustrative__ __Problem__. To illustrate the procedure in analysis, one general problem is presented. The three bay gable frame shown in Fig. 5(a) represents a case of symmetrical framing. Under the given unsymmetrical loading there are two unknown hinge reactions at joint D. The two deflection equations necessary for the solution of H and V are obtained by using Equations 2 for the contribution of the frame members in one-half of the frame to the deflections at D. In the following equations the subscripts to Δ indicate the member number and the type of deflection.

(1) Horizontal deflection at D:

$$\Delta_{1H} = H \cdot \frac{l_1}{\cos\theta_1}\left[2\left(\frac{h}{2}\right)^2 + 2\left(f + \frac{h}{2}\right)^2\right]\frac{k_1}{I_1} + V \cdot 0 +$$

$$w l_1^2 \cdot \frac{l_1}{\cos\theta_1}\left[\frac{h}{2} - \left(f + \frac{h}{2}\right)\right]\frac{k_w}{I_1} - w l_1^2 \cdot \frac{l_1}{\cos\theta_1} \cdot \frac{f}{2g}\left[\frac{h}{2} - \left(f + \frac{h}{2}\right)\right]\frac{k_1}{I_1}$$

$$\Delta_{2H} = 2H \cdot \frac{l_2}{\cos\theta_2} \cdot f^2 \cdot \frac{k_2}{I_2} + V \cdot 0 + w \cdot 0$$

$$\Delta_{3H} = 2Hh\left(\frac{h}{2}\right)^2 \cdot \frac{k_3}{I_3} + V \cdot 0 + w l_1^2 \cdot \frac{h^3}{4g} \cdot \frac{k_3}{I_3}$$

$$\Delta_{4H} = 2Hh\left(\frac{h}{2}\right)^2 \cdot \frac{k_4}{I_4} + V \cdot 0 - w l_1^2 \cdot \frac{h^3}{4g} \cdot \frac{k_4}{I_4}$$

$$\Delta_{1H} + \Delta_{2H} + \Delta_{3H} + \Delta_{4H} = \Delta_H = 0 \qquad (3)$$

(2) Vertical deflection at D:

$$\Delta_{1V} = V \cdot \frac{l_1}{\cos\theta_1} \left[2\left(\frac{l_2 h}{2g}\right)^2 + 2 l_2^2 \left(1 - \frac{h}{2g}\right)^2 \right] \frac{k_1}{I_1} + H \cdot 0 +$$

$$w l_1^2 \cdot \frac{l_1}{\cos\theta_1} \left[\frac{l_2 h}{2g} - l_2 \left(1 - \frac{h}{2g}\right) \right] \frac{k_w}{I_1} + w l_1^2 \cdot \frac{l_1}{\cos\theta_1} \cdot \frac{f}{2g} \left[\frac{l_2 b}{2g} - l_2 \left(1 - \frac{h}{2g}\right) \right] \frac{k_1}{I_1}$$

$$\Delta_{2V} = 2 l_2^2 \cdot \frac{l_2}{\cos\theta_2} \cdot \frac{k_2}{I_2} + H \cdot 0 + w \cdot 0$$

$$\Delta_{3V} = 2Vh\left(\frac{l_2 h}{2g}\right)^2 \cdot \frac{k_3}{I_3} + H \cdot 0 - w l_1^2 \frac{l_2 h^3}{4g^2} \cdot \frac{k_3}{I_3}$$

$$\Delta_{4V} = 2Vh\left(\frac{l_2 h}{2g}\right)^2 \cdot \frac{k_4}{I_4} + H \cdot 0 - w l_1^2 \cdot \frac{l_2 h^3}{4g^2} \cdot \frac{k_4}{I_4}$$

$$\Delta_{1V} + \Delta_{2V} + \Delta_{3V} + \Delta_{4V} = \Delta_V = 0 \qquad (4)$$

Each of these two final equations (Eqs. 3 and 4) contain terms in H, V and the real load. These two simultaneous equations may now be solved to obtain H and V at joint D. After finding these forces at D, the design moments may be solved directly.

Development of Simplified Method. The final equations from the preceeding article may be reduced to the following form.

$$\Delta_H = \alpha H + \beta V + Q = 0 \qquad (3)$$

$$\Delta_V = \alpha' H + \beta' V + Q' = 0 \qquad (4)$$

In these equations the symbols α and β indicate the total coefficients of the corresponding unknown, H or V. Q is the total real load factor or coefficient as applied to each equation. The prime marks (') denote the deflection equation to which the coefficient belongs.

In evaluating α, β and Q there are certain factors which depend on the properties of the individual member and other factors which depend only on the dimensional characteristics of the frame. These factors are now separated. The factor C depends on the properties of the individual member and is expressed as:

$$C_H = k \frac{l}{I} h \qquad \text{for equation (3)}$$

$$C_V = k \frac{l}{I} l_2 \qquad \text{for equation (4)}$$

In these equations, k is the same k as used in Equations (2) of the article on the deflection equations; it depends on the shape of the member. l is the length of the member, I is the moment of inertia of the small end of a wedge member, h is the height of the column (See Fig. 5a), and l_2 is half the bay span (See Fig. 5a). Each different member used in the frame will

have its own C values.

The factors r, s and p depend only on the dimensional characteristics of the frame. r is the factor associated with H. There will be a separate r factor for each member of the frame depending on the contribution of that member to H as determined only from the geometry of the frame. Similarly, s is the factor associated with V and p is the factor associated with the external real loads.

α, β and Q are now evaluated from the following equations in which the subscripts indicate the member involved (Fig. 5a).

$$\alpha = h\left[C_1 r_1 + C_2 r_2 + C_3 r_3 + C_4 r_4\right]$$

$$\beta = l_2\left[C_1 s_1 + C_2 s_2 + C_3 s_3 + C_4 s_4\right]$$

$$Q = w_1 l_1^2\left[C_1 p_1 + C_2 p_2 + C_3 p_3 + C_4 p_4\right]$$

The factors are simplified by removing h, l_2 or wl_1^2, as appropriate, from the items in parentheses. Similar equations may be written for α', β' and Q'.

Tables may now be set up which greatly simplify the solution of the problem. For any given framing the r, s and p can be determined by following the general procedure of solution as previously outlined. Once these coefficients are determined, the deflection equations can be written directly. Mr. Amirikian has evaluated and tabulated these coefficients. The tables are arranged in a form to assist the designer in writing the deflection equations. The tables giving these coefficients for different methods of cutting the frame are in the Appendix.

Only those tables which apply to gable frames are included in this work. This is admittedly a brief explanation of the development of Mr. Amirikian's method.

Illustrative Problem Using Simplified Method. The method is probably best illustrated in this brief treatment by outlining the analysis of the frame in Fig. 5(a) using the method. A step by step procedure is given.

1. Select trial members for the frame and obtain the k and k_w factors from the appended tables.

2. Cut the frame at hinge D. This means that the r, s and p factors will be obtained from the appended tables labled "Horizontal Deflection at Joint 2" and "Vertical Deflection at Joint 2".

3. Because of the extent (only three bays) and loading of the frame, only the following columns from those tables will be used: hH_2 , l_2V_2 , $w_1l_1^2$. Lay out work tables similar to the appended tables but with only the above three columns for coefficients.

4. Compute the C_H and C_V values for each member and enter them in the C columns of the work tables. Each member may have its own C. Relative values of C may be used for convenience.

5. Compute the r values from the tabulated formulae in the hH_2 columns of the appended tables and enter them in the work tables. The s values are obtained from the l_2V_2 columns, and the p values from the $w_1l_1^2$ columns in similar fashion.

6. The deflection equations are Equations (3) and (4),

which are repeated with the subscript 2 applied to H and V for convenience.

$$\Delta_H = \alpha H_2 + \beta V_2 + Q = 0 \qquad (3)$$

$$\Delta_V = \alpha' H_2 + \beta' V_2 + Q' = 0 \qquad (4)$$

7. α is evaluated by multiplying each r by its corresponding C_H and summing up the products in the table for horizontal deflections. β and Q are similarly evaluated.

8. α' is evaluated by multiplying each r' by its corresponding C_V and summing up the products in the table for vertical deflections. β' and Q' are similarly evaluated.

9. Equations (3) and (4) are solved to obtain H_2 and V_2.

10. The design moments may now be written for the entire frame and the selected members checked for allowable stresses.

11. If any member is unsatisfactory, a new member is selected. When a new member is substituted, the only items changed in the work tables are the C and k values for the member. This feature of the method makes refinement of the design a relatively simple process.

3-5. Member Design.

Wedge-beam members were used throughout all of the frames. See Fig. 1 and Fig. 2. Further, all of the wedge-beam members were selected to be fabricated from standard rolled WF sections as listed in the A.I.S.C. Steel Construction Manual. Since an objection to the use of wedge-beam framing has been the cost of fabricating the members, it was felt that standard rolled WF sections would be used in practice to keep the

fabrication costs to a minimum.

With reference to Fig. 1, the value of e was taken as two inches wherever possible. This value of e was recommended as the minimum which allowed reasonable ease of welding when the two parts of the cut WF section were fabricated into a wedge-beam section. The required section modulus of any member in an articulated wedge-beam frame varies from zero at the tapered or small end to a maximum at the large end. Therefore, the most efficient member and the lightest satisfactory member can be obtained by keeping e to a minimum. This has the effect of giving a very small section modulus at the small end and the maximum possible section modulus for the section at the large end. It was not always possible to use an e of two inches. The factors other than the zero moment which had to be considered at the small end of the member were the axial load and the transverse shear. With the larger spans these factors made it necessary to use values of e greater than two inches, but the minimum e which would give a satisfactory member was used in these cases.

In checking a member for allowable stresses the specifications of the A.I.S.C. Steel Construction Manual were followed. Column members were checked at the large end, mid-height and small end. Roof members were checked at the two ends. This was done because of the way the moment diagrams for the members varied. See Fig. 2, which shows typical column and roof member moment diagrams. The moment diagram for the column members varied in a straight line from the maximum to zero,

while the supplied section modulus varied along a curve (Fig. 1). Thus a selected column member might have been satisfactory at both ends yet overstressed at the midpoint. The moment diagrams for the roof members dropped off rapidly from a maximum at the large end. They then varied over a range of relatively small moments to zero at the small end. Thus the two ends of a roof member were its critical points.

The general procedure of member selection was much the same as for any indeterminate structure containing unlike, non-prismatic members. Initial members were selected and the frame was analyzed on the basis of those members. The members were then checked for stresses as discussed above. The originally selected members were replaced by new members whenever it was felt that the new members would improve the material economy of the frame or when an original member was found to be overstressed. After any change in members it was, of course, necessary to analyze the frame again. As was pointed out in Article 3-4, this process was relatively simple because of the tabular method of analysis used and because only a few terms of the table were changed. This general process was repeated until the final frame was arrived at. It has been mentioned in Article 3-3 that the members were only selected and detailed designs were not done.

3-6. Results.

The results of this work are shown on Graphs 1 and 2. As was explained in Article 3-2, the results are best shown on Graph 2 which was obtained from Graph 1. Graph 1 is a plot of Total Span in feet vs Weight of Framing Material in pounds per

square foot of floor area. Graph 2 is a plot of Number of Equal Span Bays vs Weight of Framing Material in pounds per square foot of floor area.

The ordinates of both graphs include only the weight of the bay framing itself since economy of framing material was the main interest in this work. This was felt to be further justifiable for two reasons. First, it was assumed that the weight of the roofing and siding expressed in terms of pounds per square foot of floor area would be nearly enough equal for all the frames to have a negligible effect on the shape of the curves. Second, the effects of changing the span per bay and the number of bays were emphasized by considering only the frame weight in the curves.

3-7. Discussion of Curves.

Since the final curves of Graph 2 depend entirely upon the curves of Graph 1, only Graph 1 need be considered in a discussion of the accuracy of the curves. After all the points on Graph 1 were plotted, it seemed that the most probable form of curve for each set of points was a straight line, and a straight line was used through each set. Because the choice of members was limited to those which could be fabricated from regularly rolled WF sections, it was impossible to design all members to the same efficiency. This was considered to partially explain the deviations from a straight line among the points. Other sources of error may also have contributed to the deviations. However, it was felt that a straight line did represent the true shape of the curves, and that the curves

were accurate within reason.

3-8. Analysis of Curves.

GRAPH 2 INDICATES THAT FOR ANY GIVEN TOTAL SPAN THE ECONOMY OF FRAMING MATERIAL INCREASES AS THE NUMBER OF EQUAL SPAN BAYS IS INCREASED, FOR A SINGLE STORY GABLE FRAME WHICH UTILIZES ARTICULATED WEDGE-BEAM FRAMING.

The curves of Graph 2 show specifically that for any given total span over eighty feet, with a column height of twenty feet, an economy of framing material is effected with each increase in the number of equal span bays up to at least four, for the type of structure considered. However, the consistent shape of the curves points to the generalized statement made at the begining of this article. There is one obvious limitation to the generalized statement. The curves show that there is an increased economy with each increase in the number of bays up to the point where the column height is equal to the span per bay. Thus for a sixty foot total span, with twenty foot columns, the best economy shown is with three twenty foot bays. For an eighty foot total span, with twenty foot columns, the best economy shown is with four twenty foot bays. The members used for the exterior columns in the designs for the twenty foot span per bay structures were very near the lightest members available which make satisfactory columns. Also because of the unsupported length of the interior columns, these members were actually heavier than the exterior columns and roof members, though the interior columns should theoret-

ically be the lightest members in the structure. Thus there is obviously a lower limit below which the span per bay cannot be reduced and still obtain increasing economy. From a standpoint of frame material economy, it seems that the span per bay should not be less than the column height.

This line of reasoning is further born out by the fact that as the number of bays is increased the percentage increase in economy for each additional bay decreases. The law of diminishing returns definitely applies. The following table to illustrate this gives average values from all of the curves on Graph 2. No curve varied more than five percent from these average values.

Increase from	Weight saved by increase (percent of larger weight)
1 to 2 bays	47%
2 to 3 bays	29%
3 to 4 bays	19.5%

From the discussion so far, it is seen that points for five equal span bays would add considerably to the value of the curves, as was mentioned in Article 3-2. Such points would definitely help to establish the lower limit of each curve. Points for structures using a fifteen foot span per bay would, of course, be brought in with the five bay structures to add to the value of the new points.

Only economy of framing material has been considered in this work. For any given structure the overall economy must be considered in its design. As was pointed out, the weight of roofing and siding material for a given total span, for the

structures designed, was nearly independent of the number of bays used. The only factor changing this quantity is the slight changes in roof slope. Thus from a purely material standpoint, the most economical structure of the type considered in this work is the one with spans per bay approximately equal to the column height. However, in the overall economy the costs of fabrication and erection must be considered. The evaluation of these items are beyond the scope of this work, and are only mentioned to point out the other phases of the problem. In any practical design the two phases must be balanced.

3-9. Suggestions for Further Study.

It is pointed out in Section II that Mr. Amirikian's concept of articulated wedge-beam framing is new, and that there are many possibilities for further investigations in connection with it. Along the lines of this work, a similar project could be done for the various systems of regular rigid framing. Such a project using the same problem conditions as this work would provide a basis for economy comparisons between regular rigid framing and articulated wedge-beam framing over a range of spans and bay arrangements. The project would also check the extent to which the results of this work apply to other types of construction.

IV. COMPUTATIONS

This section is divided into six articles. The first four articles cover the design computations for the single bay, two bay, three bay and four bay structures respectively. Each article contains a summary of the computations for the structures designed in that article. Detailed computations are presented for the forty foot span per bay structure for each article too illustrate the complete procedure for the structures in that article. In the detailed computations items which merely duplicate previous work or methods are omitted, and only the results for those items are indicated. An attempt has been made to reduce needless detail to a minimum.

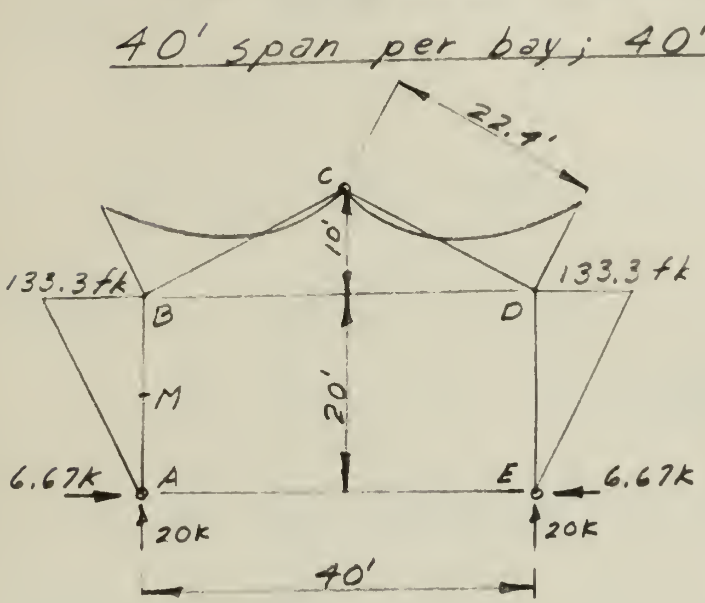
Article 5 is a tabulation of the data used to plot Graph 1.

Article 6 is a tabulation of some properties of wedge-beam members used. This tabulation was computed for use in this work as a supplement to the similar section of the A.I.S.C. *Steel Construction Manual*. The method of computing items for this table is shown in the sample computations of Article 4-1. The tabulation does not cover all of the members listed in the A.I.S.C. *Manual*. It does not include many items given in the *Manual* for each WF section. Some of the unlisted items are unchanged when the regular WF section is converted into a wedge-beam section. The *Manual* and the tabulation were used together in the design work. It is not recommended that this tabulation be used blindly in other works without full realiza-

tion that the table was constructed to slide rule accuracy only. It is presented because of its possible value as a form and a check for future work.

4-1. Single Bay Structures.

40' span per bay; 40' total span.



Try 14WF34 for all members. Column design controls.

Section Properties

I & S

Large End - $2(12.00-.45) = 2\times 11.55 = 23.1''$ web $= 295$

$24-.45 = 23.55''$ flanges $= 277\times 3.06 = \underline{847}$

$I = 1142\ in^4$

$$S = \frac{1142}{12} = 95.3\ in^3$$

Small End - $I_{(approx)} = \frac{3.06\times 16}{2} = 24\ in^4$

A & r_{xx} (r_{yy} assumed unchanged.)

Large End - $A = 2\times 3.06 + 23.1\times .287 = 6.12 + 6.63 = 12.75$ si.

$$r_{xx} = \sqrt{\frac{1142}{12.75}} = 9.5\ in.$$

Small End - $A = 6.12 + 3.10\times .287 = 6.12 + .89 = 7.01$ si.

$$r_{xx} = \sqrt{\frac{24.0}{7.01}} = 1.88\ in.$$

Stresses

@ B $f_b = \frac{133.3 \times 12}{95.3} = 16.8\text{ksi}$ $\quad F_B = 20\text{ksi}$

$f_a = \frac{20}{12.75} = 1.57\text{ ksi}$ $\quad \frac{l}{r_{xx}} = \frac{20 \times 12}{9.5} = 26$ $\quad F_A = 16.67\text{ ksi}$

$$\frac{f_a}{F_A} + \frac{f_b}{F_B} = \frac{1.57}{16.67} + \frac{16.8}{20} = .095 + .840 = .935$$

@ M $f_b = \frac{133.3 \times 12}{48.5 \times 2} = 16.5\text{ksi}$ $\quad F_B = 20\text{ ksi}$

$f_a = \frac{20}{10.0} = 2.0\text{ ksi}$ $\quad \frac{l}{r_{xx}} = \frac{20 \times 12}{5.83} = 42$ $\quad F_A = 16.14\text{ksi}$

$$\frac{f_a}{F_A} + \frac{f_b}{F_B} = \frac{2.0}{16.14} + \frac{16.5}{20} = .124 + .825 = .949$$

@ A $f_a = \frac{20}{7.01} = 2.86\text{ ksi}$ $\quad \frac{l}{r_{xx}} = \frac{20 \times 12}{1.88} = 128$ $\quad F_A = 9.04\text{ksi}$

shear allowed = 13×4×.287 = 14.9 k $\quad$ actual = 6.67 k

Final members all members - 14W=34

Weight

columns - $\quad 2 \times 20 \times 34 = 1360$

roof members - $\quad 2 \times 22.4 \times 34 = \underline{1524}$

$\quad\quad 2884$

$$\frac{2884}{20 \times 40} = 3.61\text{ psf of floor area}$$

60' span per bay; 60' total span.

Reactions at base of exterior column: H - 15k, V - 30k

Final members:

all members - 18WF55.

Weight: 4.74 psf

80' span per bay; 80' total span.

Reactions at base of exterior cloumn: H - 22.9k, V - 40k

Final members:

all members - 21WF68.

Weight: 5.34 psf

120' span per bay; 120' total span.

Reactions at base of exterior column: H - 51.5k, V - 60k

Final members:

all members - 30WF108 (use e of 3").

Weight: 7.39 psf

150' span per bay; 150' total span.

Reactions at base of exterior column: H - 70.4k, V - 75k

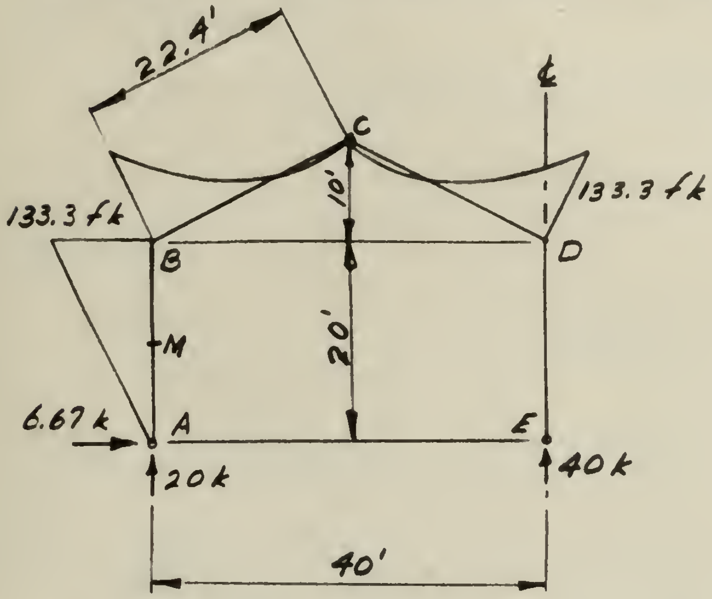
Final members:

columns - 33WF141 (use e of 5")
roof members - 33WF130 (use regular e of 2").

Weight: 8.62 psf

4-2. Two Bay Structures.

40' span per bay; 80' total span.



Try 14WF34 for exterior columns & roof members.

Stresses

@B $f_b = \frac{133.3 \times 12}{95.3} = 16.8$ ksi $\quad F_B = 20$ ksi

$f_a = \frac{20}{12.75} = 1.57$ ksi $\quad \frac{l}{r_{xx}} = 26 \quad F_A = 16.67$ ksi

$\frac{f_a}{F_A} + \frac{f_b}{F_B} = \frac{1.57}{16.67} + \frac{16.8}{20} = .095 + .840 = .935$

@M $f_b = \frac{133.3 \times 12}{48.5 \times 2} = 16.5$ ksi $\quad F_B = 20$ ksi

$f_a = \frac{20}{10.0} = 2.0$ ksi $\quad \frac{l}{r_{xx}} = 42 \quad F_A = 16.14$ ksi

$\frac{f_a}{F_A} + \frac{f_b}{F_B} = \frac{2.0}{16.14} + \frac{16.5}{20} = .124 + .825 = .949$

@A $f_a = \frac{20}{7.01} = 2.86$ ksi $\quad \frac{l}{r_{xx}} = 128 \quad F_A = 9.04$ ksi

shear allowed = 13 × 4 × .287 = 14.9k $\quad$ actual = 6.67k

Try 6B20 for interior column.

$\frac{l}{r} < 200 \quad \therefore$ minimum $r_{yy} = \frac{15 \times 12}{200} = 0.90$

@E $f_a = \frac{40}{5.33} = 7.5$ ksi; $\frac{l_y}{r_{yy}} = \frac{180}{1.50} = 120$ $\frac{l_x}{r_{xx}} = \frac{240}{1.82} = 132$

$F_A = 8.60$ ksi

Final members

Exterior columns & roof members = 14WF34

Interior column = 6B20

Weight

columns -	$2 \times 20 \times 34$	$= 1360$
	20×20	$= 400$
roof members -	$4 \times 22.4 \times 34$	$= 3050$
		4810

$\frac{4810}{20 \times 80} = 3.01$ psf of floor area.

20' span per bay; 40' total span.

Reactions at base of exterior column: H - 2k, V - 10k

Final members:

exterior columns and roof members - 12B14
interior column - 6B15.5

Weight: 1.87 psf

60' span per bay; 120' total span.

Reactions at base of exterior column: H - 15k, V - 30k

Final members:

exterior columns and roof members - 18WF55
interior column - 8WF28

Weight: 4.04 psf

80' span per bay; 160' total span.

Reactions at base of exterior column: H - 22.9k, V - 40k

Final members:

exterior columns and roof members - 21WF68
interior column - 8WF35

Weight: 4.70 psf

120' span per bay; 240' total span.

Reactions at base of exterior column: H - 51.5k, V - 60k

Final members:

exterior columns and roof members - 30WF108 (use e of 3")
interior column - 8WF48

Weight: 6.69 psf

150' span per bay; 300' total span.

Reactions at base of exterior column: H - 70.4k, V - 75k

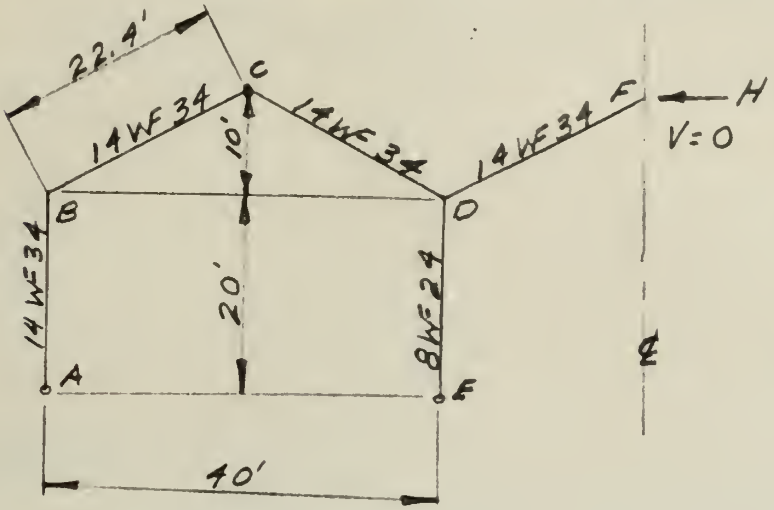
Final members:

exterior columns - 33WF141 (use e of 5")
roof members - 33WF130
interior column - 8WF67

Weight: 7.90 psf

4-3. Three Bay Structures.

40′ span per bay; 120′ total span.



Assumed members and force at cut joint. Use "Horizontal Deflection at Joint 2" Table.

k factors

14WF34 - $24=4(1+c)$ $c=5$ $k=.01800$ $k_w=.00563$

8WF24 - $c:2$ $k=.05868$ $k_w=.01966$

Member	$^*C_H=k\frac{l}{I}h$	H-V Coeff. $hH_2=20H_2$	Load Coefficient $W_1l_1^2=400$	 $W_2l_2^2=400$	 $W_3l_3^2=400$
Beam 1	1.68	1.25	−.072	−.417	
2	1.68	.5		−.313	
3	1.68	1.25		−.417	−.072
Column 0-1	1.50	.25	.167	−.083	
1-2	5.65	.25	−.167	.083	
2-3	5.65	.25		.083	−.167
3-4	1.50	.25		−.083	.167

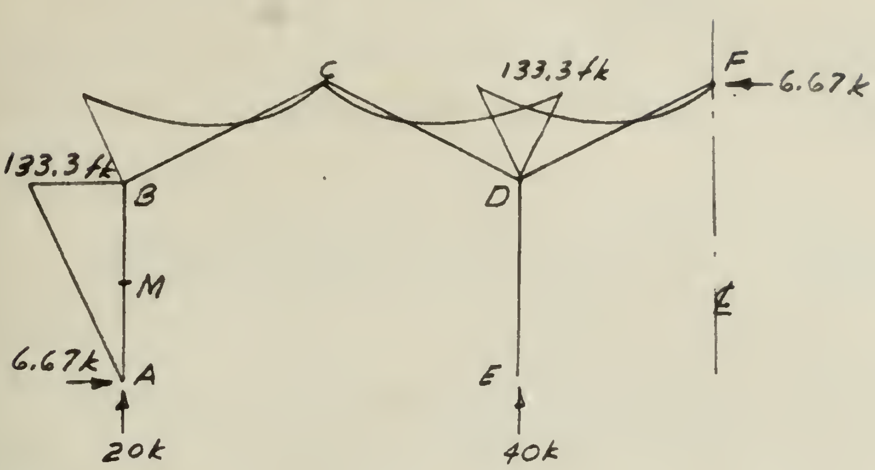
* Relative values of C.

$$20H_2(8.61)+400(-.814-1.237-.814)=0$$

$$H_2=\frac{400\times2.865}{20\times8.61}=+6.67\text{ k}$$

Note: Use of interior column member lighter than 8WF24 introduces moment in that column, which the light member will not carry.

Final Reactions & Moments.



Stresses

Checks for 14WF34 exterior column and roof members same as in Articles 4-1 and 4-2. Member O.K.

8WF24 interior column (small end critical):

@E $f_a = \frac{40}{5.98} = 6.69$ ksi $\frac{l_Y}{r_{YY}} = \frac{180}{1.61} = 112$ $\frac{l_x}{r_{xx}} = \frac{240}{1.87} = 129$

$F_A = 8.93$ ksi

Final Members.

Exterior columns and all roof members - 14WF34

Interior columns - 8WF24

Weight.

columns -	2×20×34	= 1360
	2×20×24	= 960
roof members -	6×22.4×34	= 4570
		6890

$\frac{6890}{20\times120} = 2.87$ psf of floor area.

20' span per bay; 60' total span.

Forces at cut joint: H - 1.50k, V - 0

Reactions at base of exterior column: H - 1.75k, V - 9.4k

Final members:

exterior columns and roof members - 12B14
interior columns - 6B15.5

Weight: 1.77 psf

60' span per bay; 180' total span.

Forces at cut joint: H - 15.5k, V - 0

Reactions at base of exterior column: H - 15.25k, V - 30.2k

Final members:

exterior columns and roof members - 18WF55
interior columns - 8WF28

Weight: 3.82 psf

80' span per bay; 240' total span.

Forces at cut joint: H - 24.4k, V - 0

Reactions at base of exterior column: H - 23.6k, V - 40.7k

Final members:

exterior columns - 21WF73
roof members - 21WF68
interior columns - 8WF35

Weight: 4.54 psf

100' span per bay; 300' total span.

Forces at cut joint: H -39.3k, V - 0

Reactions at base of exterior column: H - 37.5k, V - 51.25k

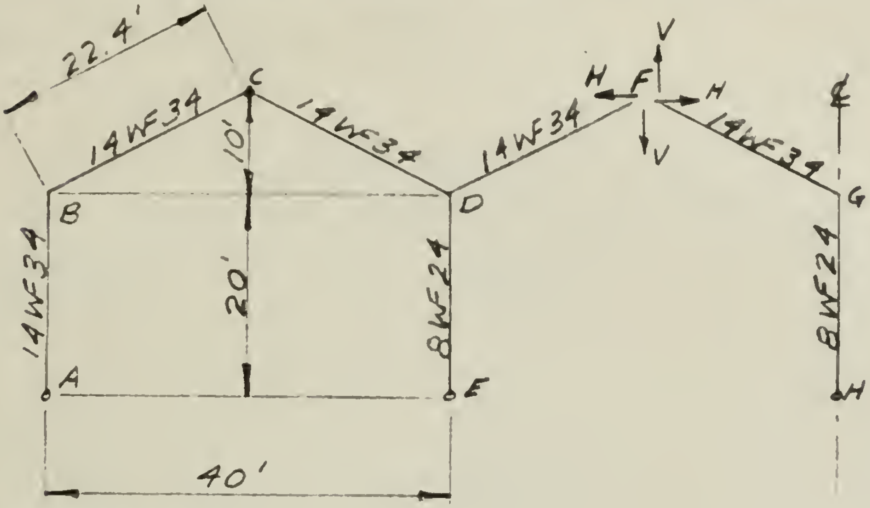
Final members:

exterior columns - 24WF100 (use e of 3")
interior columns - 8WF48
roof members-exterior bays - 24WF94
roof members-interior bays - 24WF84

Weight: 5.71 psf

4-4. Four Bay Structures.

40′ span per bay; 160′ total span.



Assumed members. Cut at Joint 2.

k factors

14WF34	$k = .01800$	$k_w = .00563$
8WF24	$k = .05868$	$k_w = .01966$

Horizontal Deflections @ Joint 2

Member		$C_H = k\frac{l}{I}h$	H-V Coeff. for ΔH_2		Load Coeff. for ΔH_2		
			$hH_2 = 20H_2$	$l_2V_2 = 20V_2$	$w_1 l_1^2 = 400$	$w_2 l_2^2 = 400$	$w_3 l_3^2 = 400$
Beam	1	1.68	1.25	-.833	-.072	-.417	
	2	1.68	.5			-.313	
	3	1.68	1.25	.833		-.417	-.072
Column	0-1	1.50	.25	-.167	.167	-.083	
	1-2	5.65	.25	.167	-.167	.083	
	2-3	5.65	.25	-.167		.083	-.167
	3-4	5.65	.25	.167		-.083	.167

$$+9.655(20H_2) + .693(20V_2) - 2.519(400) = 0$$

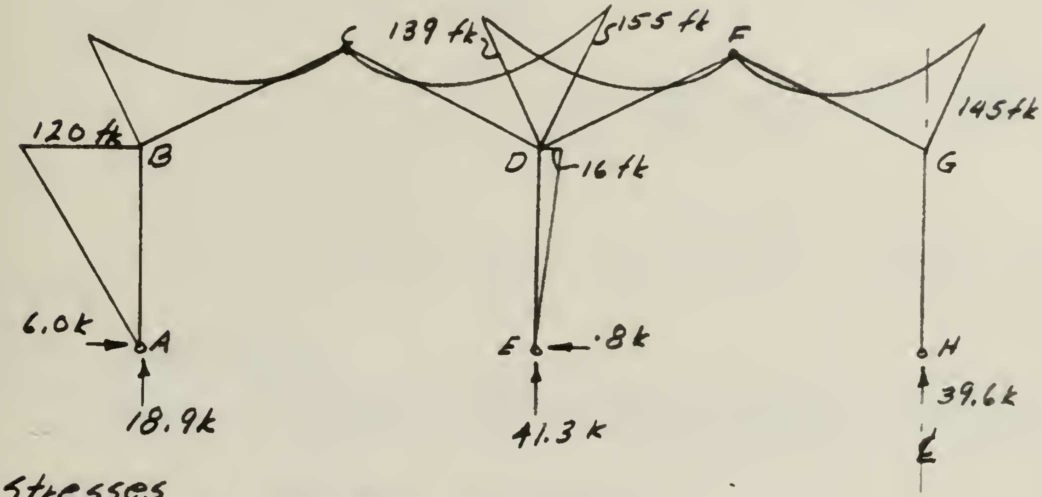
Vertical Deflections @ Joint 2

Member		$C_V = k\frac{l}{I}l_2$	H-V Coeff. for ΔV_2		Load Coeff. for ΔV_2		
			$hH_2 = 20H_2$	$l_2V_2 = 20V_2$	$w_1 l_1^2 = 400$	$w_2 l_2^2 = 400$	$w_3 l_3^2 = 400$
Beam	1	1.68	-.833	.555	.0487	.278	
	2	1.68		2.00			
	3	1.68	.833	.555		-.278	-.0487
Column	0-1	1.50	-.167	.111	-.111	.0555	
	1-2	5.65	.167	.111	-.111	.0555	
	2-3	5.65	-.167	.111		-.0555	.111
	3-4	5.65	.167	.111		-.0555	.111

$$+.693(20H_2)+7.277(20V_2)-.231(400)=0$$

$$H_2 = 5.2\ k \qquad V_2 = .14k$$

Final Reactions & Moments.



Stresses

14 WF 34 - exterior columns.

@ B $f_b = \frac{120 \times 12}{95.3} = 13.6$ ksi $\qquad F_B = 20$ ksi

$f_a = \frac{18.9}{12.75} = 1.49$ ksi $\qquad \frac{l}{r_{xx}} = 26 \qquad F_A = 16.67$ ksi

$$\frac{f_a}{F_A} + \frac{f_b}{F_B} = \frac{1.49}{16.67} + \frac{13.6}{20} = .090 + .680 = .770$$

@ A $f_a = \frac{18.9}{7.01} = 2.7$ ksi $\qquad \frac{l}{r_{xx}} = 128 \qquad F_A = 9.04$ ksi

shear allowed = 13 × 4 × .287 = 14.9 actual = 6.0 k

14 WF 34 - roof members

@ D $f_b = \frac{155 \times 12}{95.3} = 19.5$ ksi $\qquad F_B = 20$ ksi

$f_a = \frac{4.5}{12.75} = .35$ ksi $\qquad \frac{l}{r_{xx}} = \frac{22.4 \times 12}{9.5} = 29 \qquad F_A = 16.59$ ksi

$$\frac{f_a}{F_A} + \frac{f_b}{F_B} = \frac{.35}{16.59} + \frac{19.5}{20} = .021 + .975 = .996$$

8 WF 24 - interior columns (all)

@ D $f_b = \frac{16\times12}{33.3} = 5.78$ ksi $\quad \frac{Ld}{bt} = \frac{180\times11.86}{2.60} = 820 \quad F_B = 14.6$ ksi

$f_a = \frac{41.3}{7.90} = 5.23$ ksi $\quad \frac{l_y}{r_{yy}} = \frac{180}{1.61} = 112 \quad \frac{l_x}{r_{xx}} = \frac{240}{5.0} = 48$

$F_A = 10.92$ ksi

$$\frac{f_a}{F_A} + \frac{f_b}{F_B} = \frac{5.23}{10.92} + \frac{5.78}{14.6} = .479 + .396 = .875$$

@ E $f_a = \frac{41.3}{5.98} = 6.91$ ksi $\quad \frac{l_y}{r_{yy}} = \frac{180}{1.61} = 112 \quad \frac{l_x}{r_{xx}} = \frac{240}{1.82} = 129$

$F_A = 8.93$ ksi

shear allowed $= 13\times4\times.245 = 12.7$k actual $= 0.8$k

Final members

Exterior columns & all roof members - 14WF34

Interior columns (all) - 8WF24

Weight

columns - $2\times20\times34 = 1360$
$3\times20\times24 = 1440$

roof members - $8\times22.4\times34 = \underline{6100}$
8900

$\frac{8900}{20\times160} = 2.79$ psf of floor area.

20' span per bay; 80' total span.

Forces at cut joint: H - 1.4k, V - .08k

Reactions at base of exterior column: H - 1.8k, V - 9.45k

Final members:

exterior columns and roof members - 12B14
interior columns - 6B15.5

Weight: 1.72 psf

60' span per bay; 240' total span.

Forces at cut joint: H - 15.9k, V - .5k

Reactions at base of exterior column: H - 15.25k, V - 30.2k

Final members:

exterior columns and roof members - 18WF55
interior columns - 8WF28

Weight: 3.71 psf

80' span per bay; 320' total span.

Forces at cut joint: H - 26.2k, V - 1.2k

Reactions at base of exterior column: H - 24.0k, V - 40.9k

Final members:

exterior columns - 21WF73
interior columns - 8WF40
center column - 8WF35
roof members - exterior bays - 21WF73
foor members - interior bays - 21WF68

Weight: 4.58 psf

100' span per bay; 400' total span.

Forces at cut joint: H - 41.5k, V - 2.1k

Reactions at base of exterior column: H - 37.2k, V - 51.0k

Final members:

exterior columns - 24WF100 (use e of 3")
interior columns - 8WF58
center column - 8WF48
roof members - exterior bays - 24WF94
roof members - interior bays - 24WF84

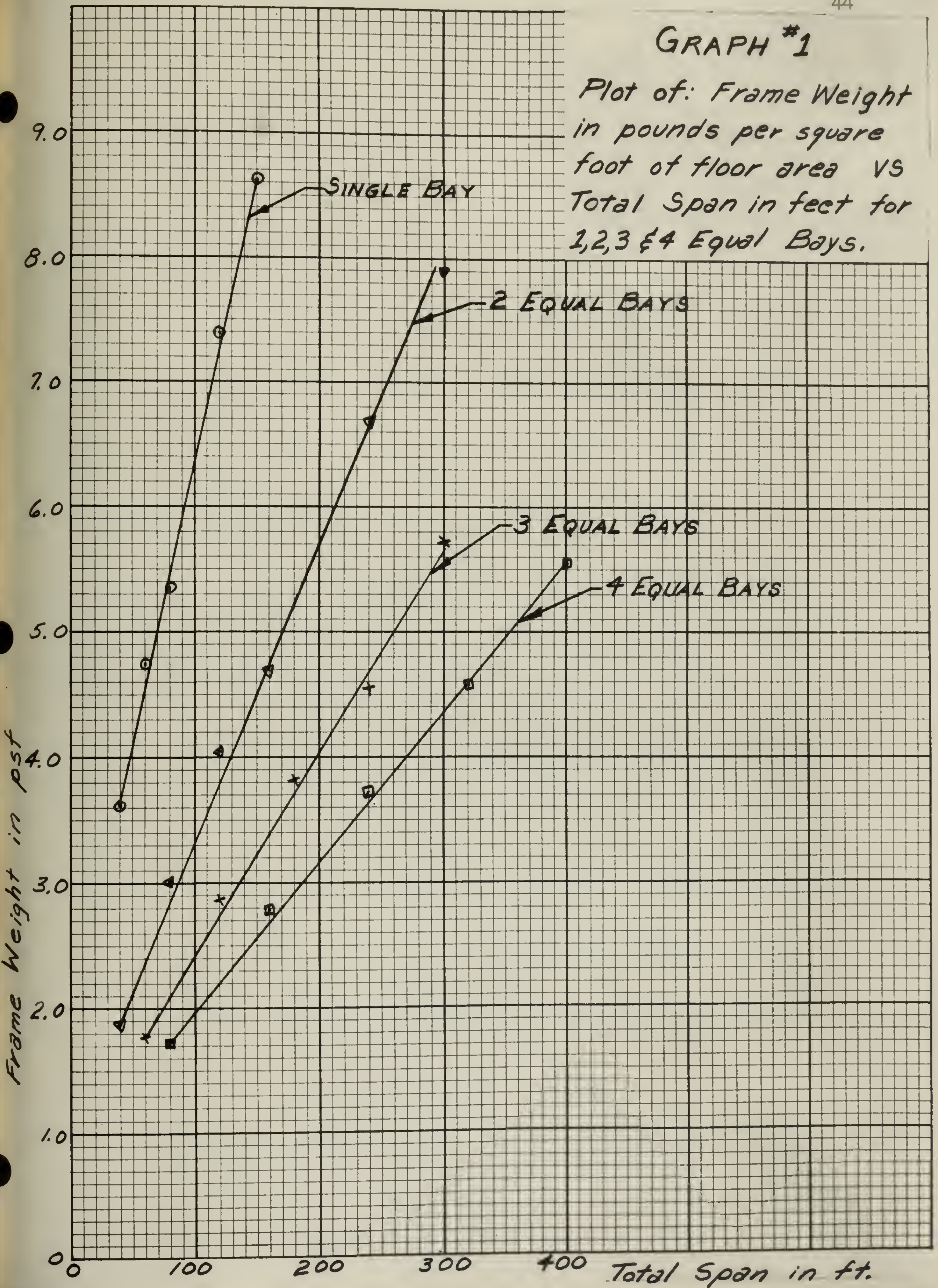
Weight: 5.55 psf

4-5. Tabulation of Weights.

No. of Bays	Span per bay	Total span	psf of floor area
1	40	40	3.61
1	60	60	4.74
1	80	80	5.34
1	120	120	7.39
1	150	150	8.62
2	20	40	1.87
2	40	80	3.01
2	60	120	4.04
2	80	160	4.70
2	120	240	6.69
2	150	300	7.90
3	20	60	1.77
3	40	120	2.87
3	60	180	3.82
3	80	240	4.54
3	100	300	5.71
4	20	80	1.72
4	40	160	2.79
4	60	240	3.71
4	80	320	4.58
4	100	400	5.55

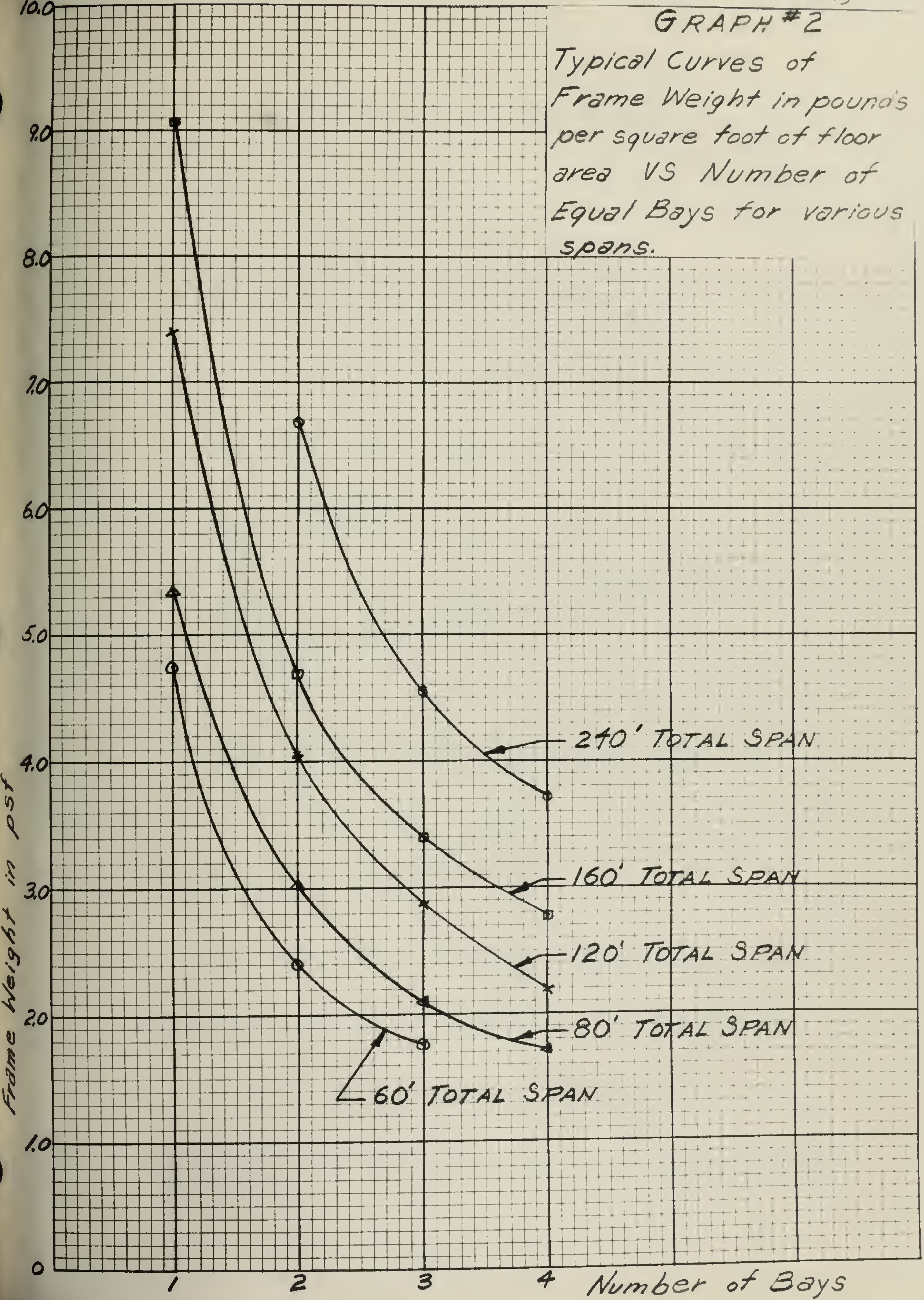
4-6. Some Properties of Selected Wedge-Beam Sections.

Nominal Size	LARGE END					SMALL END			
	Depth	Area	I_{xx}	S_{xx}	r_{xx}	Depth	Area	I_{xx}	r_{xx}
33 WF 130	62.20	54.70	29210	940	23.1	4	21.00	78.8	1.94
30 WF 124	56.32	51.40	22890	814	21.1	4	20.85	78.3	1.94
27 WF 102	50.14	40.40	14970	597	19.2	4	17.69	66.2	1.94
24 WF 94	44.58	37.92	10940	491	17.00	4	17.09	63.2	1.93
24 WF 84	44.18	34.00	9610	435	16.83	4	15.08	55.6	1.92
21 WF 73	38.48	29.10	6290	326.5	14.7	4	13.42	49.1	1.91
21 WF 68	38.26	27.20	5800	303	14.6	4	12.45	45.3	1.91
18 WF 55	32.32	21.56	3334	206.5	12.4	4	10.55	37.9	1.90
16 WF 40	28.00	15.34	1833	131	10.93	4	7.96	28.0	1.88
14 WF 34	24.00	12.75	1142	95.3	9.5	4	7.01	24.0	1.88
14 WF 30	23.72	11.36	969	81.8	9.3	4	6.03	20.6	1.85
8 WF 67	14.00	22.58	749	107.0	7.3	4	16.82	61.8	1.92
8 WF 58	13.50	19.48	607	90.0	6.7	4	14.64	53.2	1.90
8 WF 48	13.00	15.93	474	73.0	5.45	4	12.29	44.3	1.90
8 WF 40	12.50	13.31	367	58.8	5.25	4	10.21	36.05	1.88
8 WF 35	12.24	11.60	311	50.8	5.17	4	9.00	31.7	1.88
8 WF 31	12.00	10.27	263	43.8	5.09	4	7.97	27.5	1.86
8 WF 28	12.12	9.39	205	33.8	4.79	4	7.07	24.2	1.85
8 WF 24	11.86	7.90	197	33.3	5.00	4	5.98	20.8	1.87
6 WF 20	8.40	6.47	81	19.3	3.54	4	5.33	17.7	1.82
6 WF15.5	8.00	5.03	57	14.3	3.36	4	4.07	13.0	1.79
12 B 16.5	20.00	6.70	350	35.0	7.25	4	3.02	8.6	1.69
12 B 14	19.82	5.72	292	29.4	7.10	4	2.56	7.11	1.67
30 WF 108	53.64	44.50	18040	673	20.1	6	18.36	143.0	2.80
24 WF 100	42.00	37.50	10475	493	16.71	6	20.68	160.0	2.78



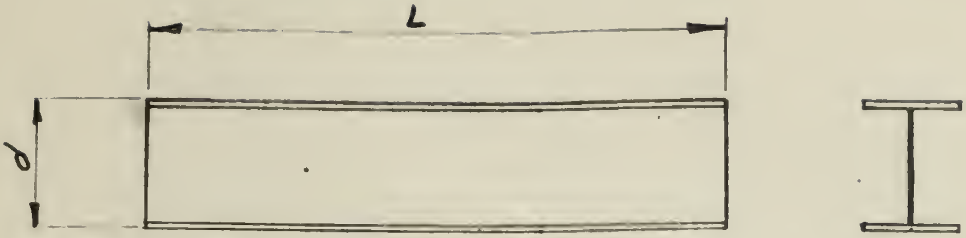
GRAPH #1

Plot of: Frame Weight in pounds per square foot of floor area VS Total Span in feet for 1,2,3 & 4 Equal Bays.

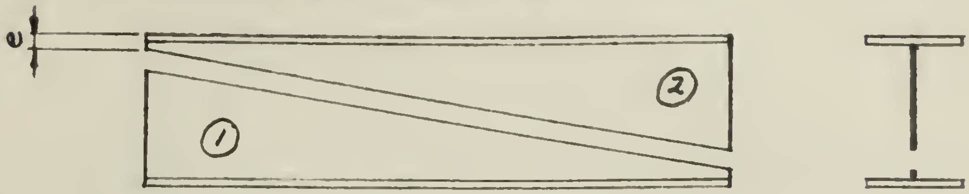


GRAPH #2

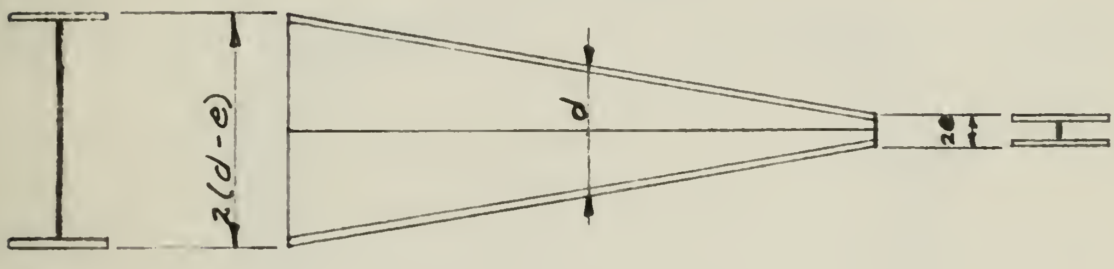
Typical Curves of Frame Weight in pounds per square foot of floor area VS Number of Equal Bays for various spans.



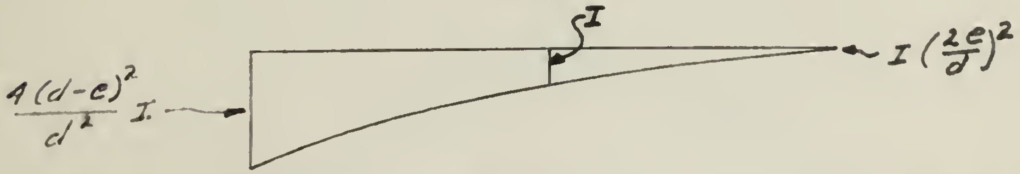
ROLLED WF SECTION



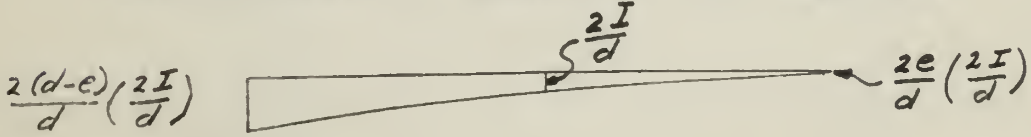
WF CUT INTO TWO WEDGE SEGMENTS



SEGMENTS 1 AND 2 WELDED TOGETHER



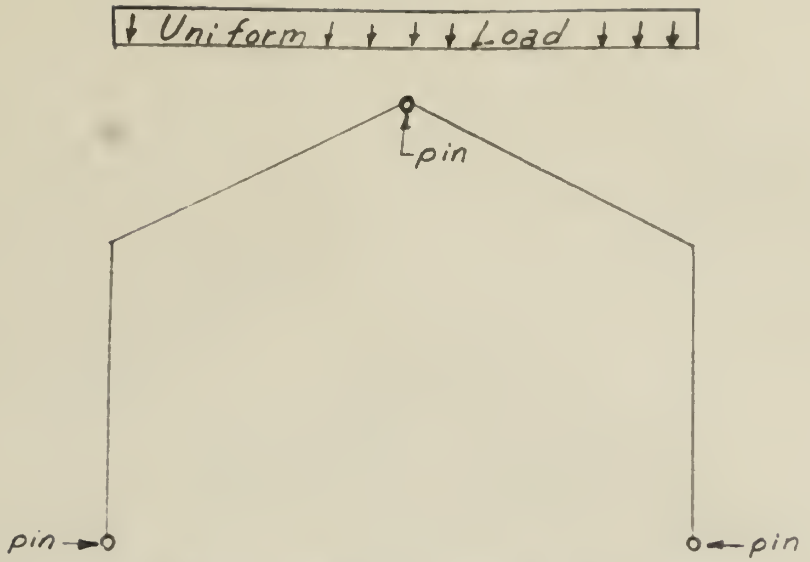
APPROXIMATE MOMENTS OF INERTIA


$\frac{2I}{d}$
$\frac{2e}{d}\left(\frac{2I}{d}\right)$
$\frac{2(d-e)}{d}\left(\frac{2I}{d}\right)$

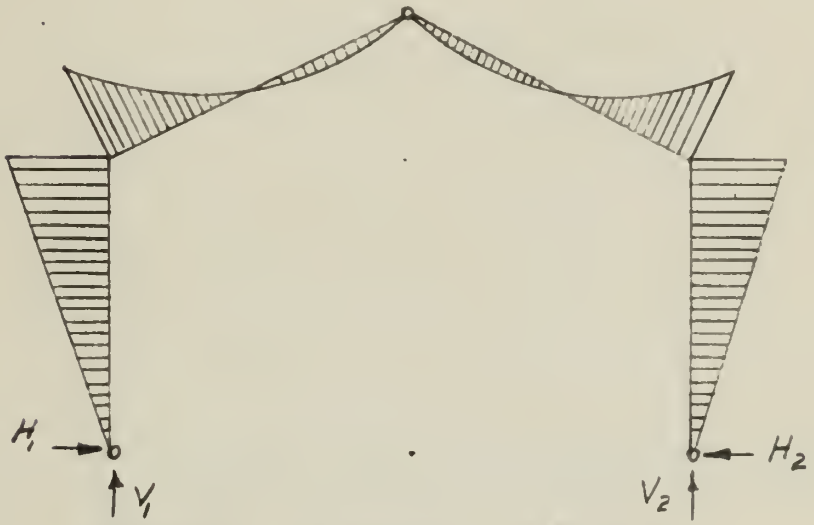
APPROXIMATE SECTION MODULI

CONSTRUCTION & PROPERTIES OF WEDGE-BEAM

FIG 1

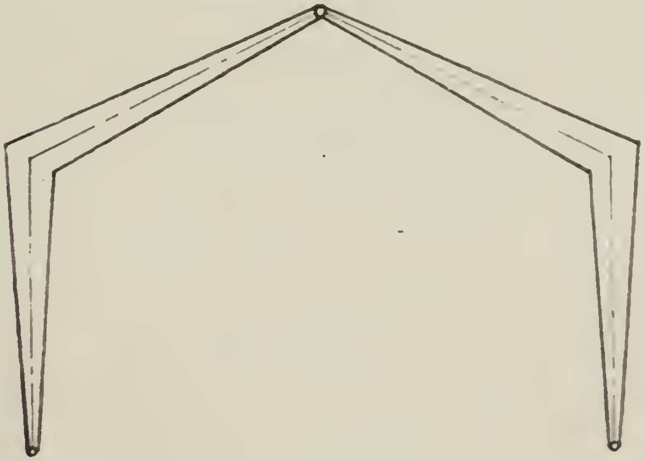


SINGLE BAY GABLE FRAME WITH LOAD



Ordinates
Shown on
Tension Side

BENDING MOMENT DIAGRAM



FRAME UTILIZING WEDGE-BEAMS

FIG 2

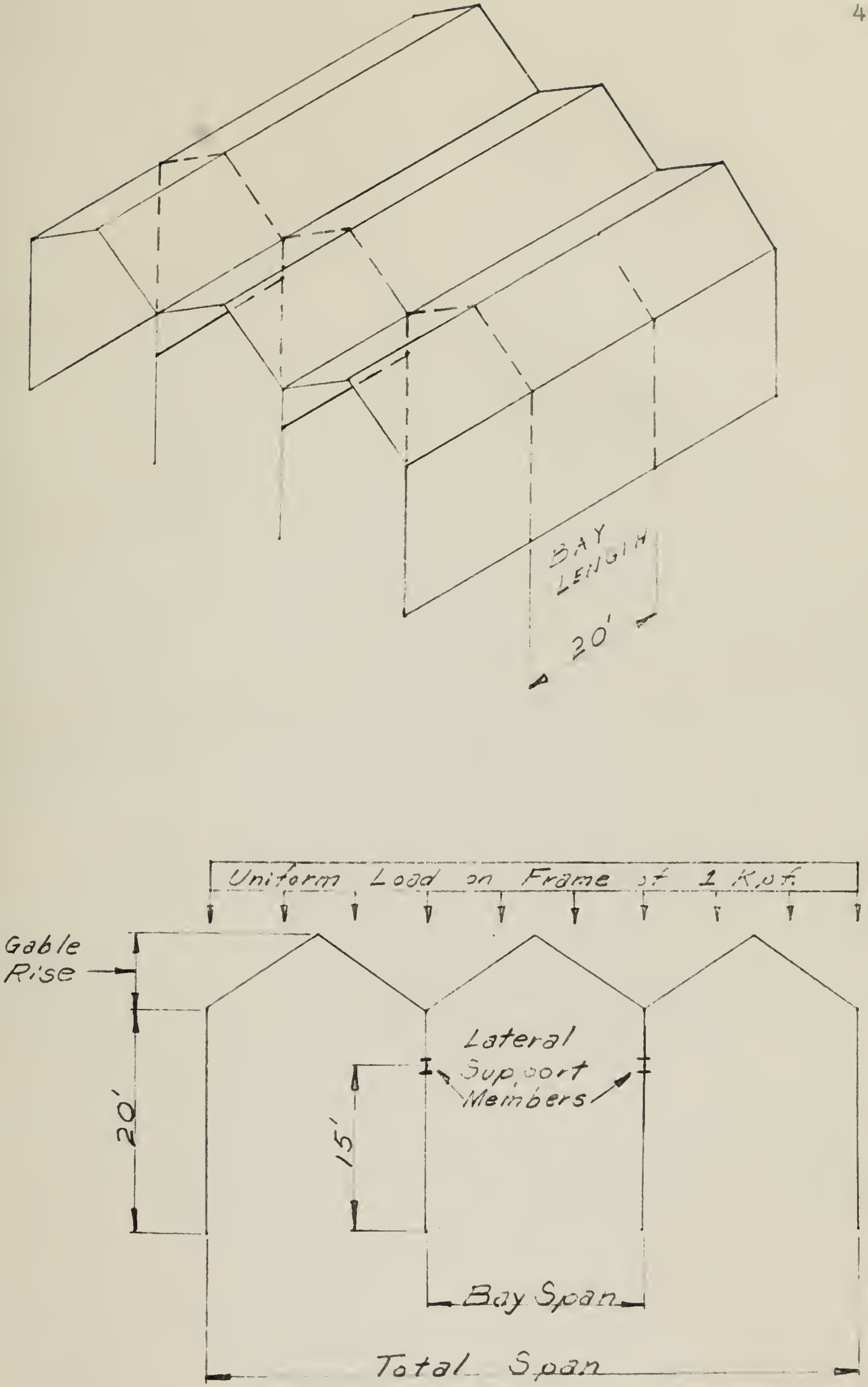

BAY LENGTH
20'
Uniform Load on Frame of 1 K.p.f.
Gable Rise
20'
15'
Lateral Support Members
Bay Span
Total Span

FIG 3

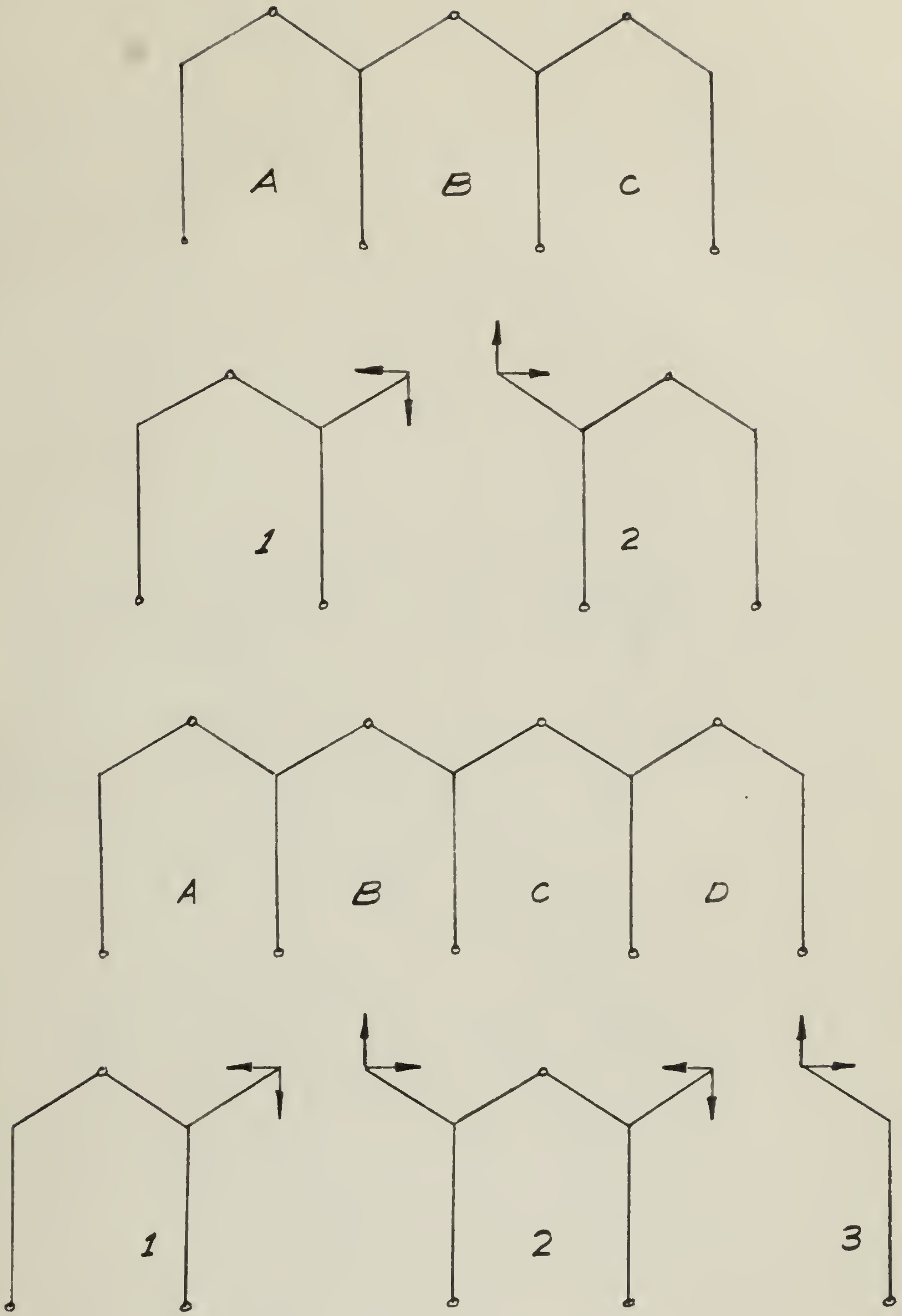


FIG 4

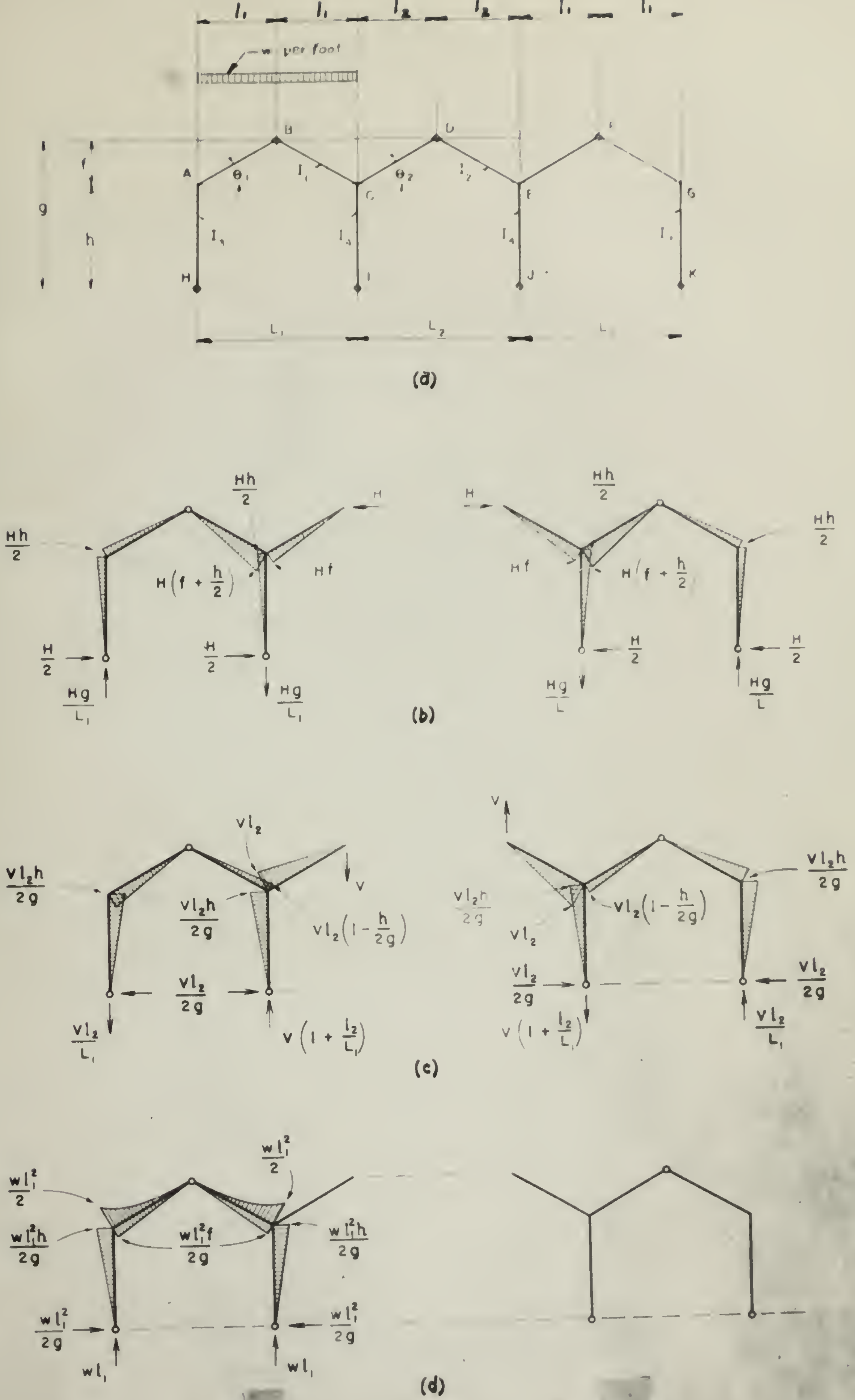

l₁ l₁ l₂ l₂ l₁ l₁
w per foot
A B C D E F G
H I J K
θ₁ θ₂
I₁ I₂ I₃ I₄
f g h
L₁ L₂ L₁
(a)
Hh/2
H
H(f + h/2)
Hf
H/2
Hg/L₁
Hg/L
(b)
Vl₂
Vl₂h/2g
V
Vl₂(1 - h/2g)
Vl₂/2g
Vl₂/L₁
V(1 + l₂/L₁)
(c)
wl₁²/2
wl₁²h/2g
wl₁²f/2g
wl₁²/2g
wl₁
(d)

FIG. 5

VI. BIBLIOGRAPHY

Amirikian, A. *Analysis of Rigid Frames.* U.S. Government Printing Office, Washington, D.C. 1942.

Amirikian, A. "Future Developments in Welded Steel Buildings." *The Welding Journal.* Vol. 27. No. 8 Aug. 1948 pp. 593-599.

Amirikian, A. "Wedge-Beam Framing." (A paper to be published). Bureau of Yards and Docks, Navy Department, Washington, D.C.

A.I.S.C. Inc. *Steel Construction* (Manual). A.I.S.C. Inc. New York, N.Y. 1946.

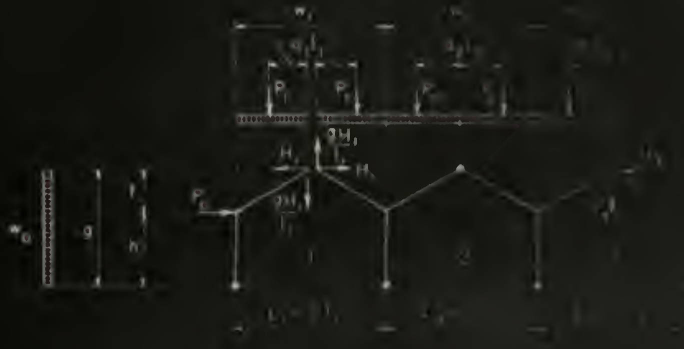
Griffiths, J. D. *Single Span Rigid Frames in Steel.* A.I.S.C. Inc. New York, N.Y. 1948.

Kinney, J. S. *Indeterminate Structures.* Rensselaer Polytechnic Institute, Troy, N.Y. 1949.

Wilber & Norris. *Elementary Structural Analysis.* McGraw Hill. New York, N.Y. 1948.

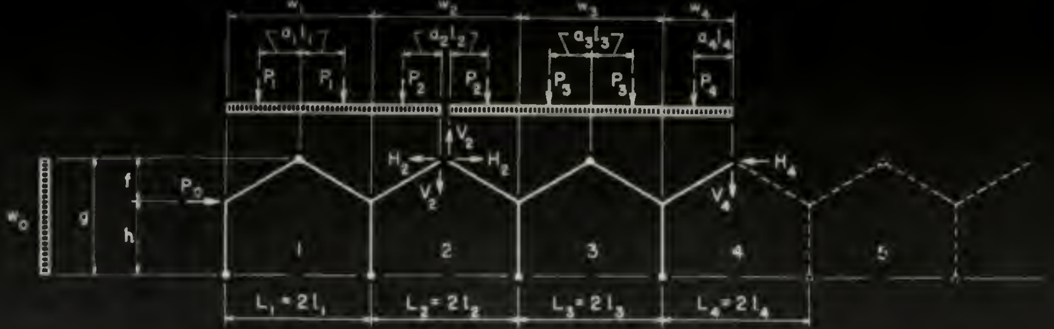
APPENDIX

The tables and forms included in the Appendix have been developed by Mr. Arsham Amirikian, Head Designer with the Bureau of Yards and Docks, U. S. Navy. The included items are only a portion of a group of tables and charts designed to assist in the analysis of articulated wedge-beam frames. The complete group of tables is included in articles now being published by Mr. Amirikian.



HORIZONTAL [illegible]

MEMBER	$C = k\frac{l}{I}h$	H-V COFF'S FOR Δ_{D1}			[illegible]								
		hH_1	hH_3	l_3V_3	hP_0	$(1-a$ [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
BEAM $1\frac{l}{r}$	C_1	$\frac{2(f^2+g^2)}{h^2}$			[illegible]	[illegible]			[illegible]	[illegible]			
$2\frac{l}{r}$	C_2	$\frac{2(f^2+g^2)}{h^2}$	$\frac{f^2-g^2}{h^2}$	$\frac{f+g}{g}$	$\frac{f^2+g^2}{gh}$		[illegible]	[illegible]			[illegible]	[illegible]	[illegible]
$3\frac{l}{r}$	C_3												
COL 0-1	C_{0-1}	1			1	$\frac{h}{g}$			[illegible]	[illegible]			
1-2	C_{1-2}												
2-3	C_{2-3}	1	$-\frac{1}{2}$	$-\frac{h}{2g}$	$\frac{h}{2g}$		[illegible]		[illegible]		[illegible]		[illegible]

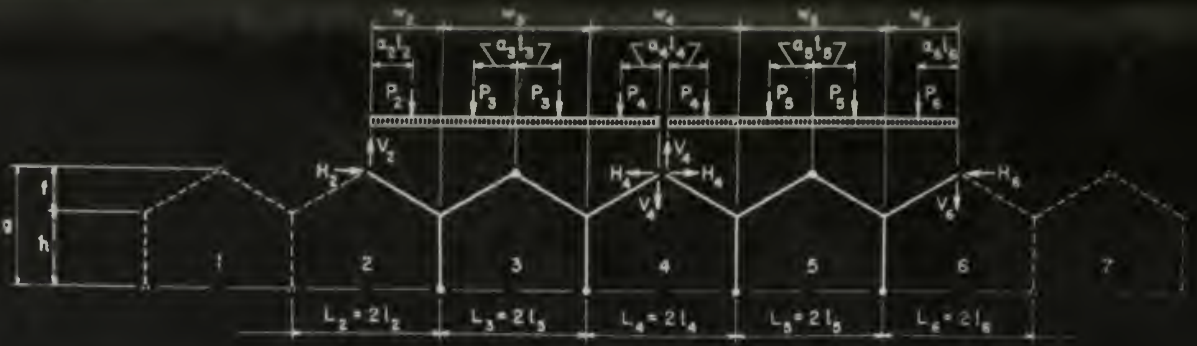


HORIZONTAL DEFLECTION AT JOINT 2

MEMBER	$C = k\frac{l}{I}h$	H-V COEFFICIENTS FOR Δ_{H2}				LOAD COEFFICIENTS FOR Δ_{H2}									
		hH_2	hH_4	l_2V_2	l_4V_4	hP_0	$(1-a_1)l_1P_1$	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	w_0g^2	$w_1l_1^2$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$
BEAM 1_r^l	C_1	$\frac{f^2+g^2}{2h^2}$		$-\frac{f^2+g^2}{2gh}$		$-\frac{f+g}{2g}$	$\frac{f^2}{gh} - \frac{fk_p}{h(1-a_1)k}$	$-\frac{f^2+g^2}{2gh}$			$\frac{flk_w}{2g^2k} - \frac{f+g}{4g}$	$\frac{f^2}{2gh} - \frac{fk_w}{hk}$	$-\frac{f^2+g^2}{4gh}$		
2_r^l	C_2	$\frac{2f^2}{h^2}$						$-\frac{2fk_p}{h(1-a_2)k}$					$-\frac{2fk_w}{hk}$		
3_r^l	C_3	$\frac{f^2+g^2}{2h^2}$	$\frac{f^2-g^2}{2h^2}$	$\frac{f^2+g^2}{2gh}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{2gh}$	$\frac{f^2}{gh} - \frac{fk_p}{h(1-a_3)k}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{4gh}$	$\frac{f^2}{2gh} - \frac{fk_w}{hk}$	$\frac{g^2-f^2}{4gh}$
COL. 0-1	C_{0-1}	$\frac{1}{4}$		$-\frac{h}{4g}$		$-\frac{f+g}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$\frac{h^2k_w}{2g^2k} - \frac{3h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$		
1-2	C_{1-2}	$\frac{1}{4}$		$\frac{h}{4g}$		$-\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$-\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$		
2-3	C_{2-3}	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{h}{4g}$	$\frac{h}{4g}$			$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$
3-4	C_{3-4}	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{h}{4g}$	$-\frac{h}{4g}$			$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$

VERTICAL DEFLECTION AT JOINT 2

MEMBER	$C = k\frac{l}{I}l_2$	H-V COEFFICIENTS FOR Δ_{V2}				LOAD COEFFICIENTS FOR Δ_{V2}									
		hH_2	hH_4	l_2V_2	l_4V_4	hP_0	$(1-a_1)l_1P_1$	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	w_0h^2	$w_1l_1^2$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$
BEAM 1_r^l	C_1	$-\frac{f^2+g^2}{2gh}$		$\frac{f^2+g^2}{2g^2}$		$\frac{g^2-f^2}{2g^2}$	$\frac{fk_p}{g(1-a_1)k} - \frac{f^2}{g^2}$	$\frac{f^2+g^2}{2g^2}$			$\frac{g^2-f^2}{4h^2} - \frac{flk_w}{2ghk}$	$\frac{fk_w}{gk} - \frac{f^2}{2g^2}$	$\frac{f^2+g^2}{4g^2}$		
2_r^l	C_2			/2											
3_r^l	C_3	$\frac{f^2+g^2}{2gh}$	$\frac{f^2-g^2}{2gh}$	$\frac{f^2+g^2}{2g^2}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{2g^2}$	$\frac{f^2}{g^2} - \frac{fk_p}{g(1-a_3)k}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{4g^2}$	$\frac{f^2}{2g^2} - \frac{fk_w}{gk}$	$\frac{g^2-f^2}{4g^2}$
COL. 0-1	C_{0-1}	$-\frac{h}{4g}$		$\frac{h^2}{4g^2}$		$\frac{g^2-f^2}{4g^2}$	$\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{3}{8} - \frac{hk_w}{2gk}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
1-2	C_{1-2}	$\frac{h}{4g}$		$\frac{h^2}{4g^2}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$-\frac{1}{8}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
2-3	C_{2-3}	$-\frac{h}{4g}$	$\frac{h}{4g}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$
3-4	C_{3-4}	$\frac{h}{4g}$	$-\frac{h}{4g}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$

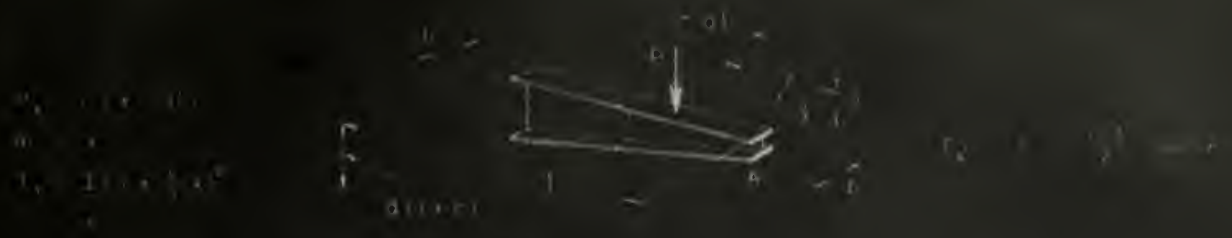


HORIZONTAL DEFLECTION AT JOINT 4

MEMBER	$C=k\frac{l}{I}h$	H-V COEFFICIENTS FOR Δ_{H4}						LOAD COEFFICIENTS FOR Δ_{H4}									
		hH_2	hH_4	hH_6	l_2V_2	l_4V_4	l_6V_6	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	$(1-a_5)l_5P_5$	$(1-a_6)l_6P_6$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$	$w_5l_5^2$	$w_6l_6^2$
BEAM $3\frac{l}{r}$	C_3	$\frac{f^2-g^2}{2h^2}$	$\frac{f^2+g^2}{2h^2}$		$\frac{f^2-g^2}{2gh}$	$-\frac{f^2+g^2}{2gh}$		$\frac{g^2-f^2}{2gh}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_3)k}$	$-\frac{f^2+g^2}{2gh}$			$\frac{g^2-f^2}{4gh}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$-\frac{f^2+g^2}{4gh}$		
$4\frac{l}{r}$	C_4		$\frac{2f^2}{h^2}$							$-\frac{2fk_p}{h(1-a_4)k}$					$-\frac{2fk_w}{hk}$		
$5\frac{l}{r}$	C_5		$\frac{f^2+g^2}{2h^2}$	$\frac{f^2-g^2}{2h^2}$		$\frac{f^2+g^2}{2gh}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{2gh}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_5)k}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{4gh}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$\frac{g^2-f^2}{4gh}$
COL. 2-3	C_{2-3}	$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{h}{4g}$	$-\frac{h}{4g}$		$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$		
3-4	C_{3-4}	$-\frac{1}{4}$	$\frac{1}{4}$		$-\frac{h}{4g}$	$\frac{h}{4g}$		$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$		
4-5	C_{4-5}		$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{h}{4g}$	$\frac{h}{4g}$			$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$
5-6	C_{5-6}		$\frac{1}{4}$	$-\frac{1}{4}$		$\frac{h}{4g}$	$-\frac{h}{4g}$			$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$

VERTICAL DEFLECTION AT JOINT 4

MEMBER	$C=k\frac{l}{I}l_4$	H-V COEFFICIENTS FOR Δ_{V4}						LOAD COEFFICIENTS FOR Δ_{V4}									
		hH_2	hH_4	hH_6	l_2V_2	l_4V_4	l_6V_6	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	$(1-a_5)l_5P_5$	$(1-a_6)l_6P_6$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$	$w_5l_5^2$	$w_6l_6^2$
BEAM $3\frac{l}{r}$	C_3	$\frac{g^2-f^2}{2gh}$	$-\frac{f^2+g^2}{2gh}$		$\frac{g^2-f^2}{2g^2}$	$\frac{f^2+g^2}{2g^2}$		$\frac{f^2-g^2}{2g^2}$	$\frac{fk_p}{g(1-a_3)k}-\frac{f^2}{g^2}$	$\frac{f^2+g^2}{2g^2}$			$\frac{f^2-g^2}{4g^2}$	$\frac{fk_w}{gk}-\frac{f^2}{2g^2}$	$\frac{f^2+g^2}{4g^2}$		
$4\frac{l}{r}$	C_4					2											
$5\frac{l}{r}$	C_5		$\frac{f^2+g^2}{2gh}$	$\frac{f^2-g^2}{2gh}$		$\frac{f^2+g^2}{2g^2}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{2g^2}$	$\frac{f^2}{g^2}-\frac{fk_p}{g(1-a_5)k}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{4g^2}$	$\frac{f^2}{2g^2}-\frac{fk_w}{gk}$	$\frac{g^2-f^2}{4g^2}$
COL. 2-3	C_{2-3}	$\frac{h}{4g}$	$-\frac{h}{4g}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{4g^2}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{h^2}{8g^2}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
3-4	C_{3-4}	$-\frac{h}{4g}$	$\frac{h}{4g}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{4g^2}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{h^2}{8g^2}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
4-5	C_{4-5}		$-\frac{h}{4g}$	$\frac{h}{4g}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$
5-6	C_{5-6}		$\frac{h}{4g}$	$-\frac{h}{4g}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$

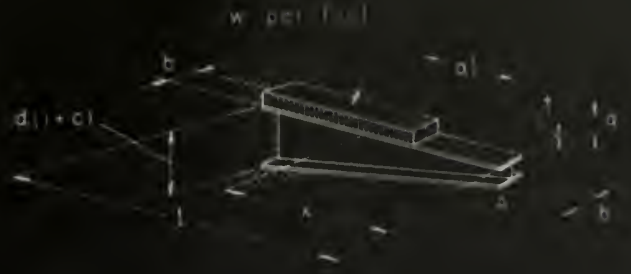


$$\Delta_A = \int_0^l \frac{M_x\, m\, dx}{EI_x} = \frac{Pl^3}{EI}\cdot\frac{1}{c^2}\left(\frac{(1-a)(2+c)}{1+c} - \frac{2+ca}{c}\log_e\frac{1+c}{1+ca}\right) = \frac{Pl^3}{EI}\cdot k_p$$

VALUES OF k_p*								
c	a							
	0.0	0.2	1/3	0.4	0.5	0.6	0.8	1.0
.0	.33333	.23467	.17284	.14400	.10417	.05933	.01867	0
.1	.28860	.20200	.14800	.12290	.08850	.06060	.01560	0
.2	.25293	.17620	.12878	.10598	.07620	.05023	.01323	0
.3	.22372	.15477	.11229	.09281	.06796	.04352	.01138	0
.4	.19953	.13731	.09921	.08181	.05671	.03808	.00989	0
.5	.17922	.12273	.08833	.07267	.05164	.03361	.00867	0
.6	.16200	.11041	.07916	.06501	.04597	.02989	.00766	0
.7	.14726	.09991	.07138	.05851	.04126	.02675	.00682	0
.8	.13451	.09087	.06470	.05294	.03724	.02408	.00611	0
.9	.12346	.08303	.05893	.04815	.03379	.02180	.00551	0
1.0	.11371	.07618	.05392	.04398	.03080	.01983	.00489	0
1.2	.09743	.06486	.04564	.03713	.02590	.01561	.00415	0
1.4	.08469	.05592	.03916	.03178	.02208	.01417	.00351	0
1.6	.07431	.04874	.03397	.02751	.01906	.01215	.00300	0
1.8	.06578	.04288	.02976	.02405	.01662	.01058	.00260	0
2.0	.05868	.03803	.02629	.02121	.01467	.00927	.00227	0
2.25	.05136	.03305	.02275	.01832	.01253	.00796	.00194	0
2.5	.04536	.02900	.01989	.01598	.01096	.00686	.00166	0
2.75	.04038	.02566	.01753	.01407	.00962	.00606	.00147	0
3.0	.03620	.02288	.01558	.01248	.00852	.00536	.00129	0
3.5	.02961	.01852	.01253	.01006	.00681	.00417	.00103	0
4.0	.02471	.01530	.01031	.00836	.00557	.00349	.00085	0
5.0	.01800	.01097	.00732	.00582	.00393	.00245	.00058	0
6.0	.01373	.00825	.00547	.00434	.00292	.00181	.00043	0
7.0	.01083	.00643	.00425	.00336	.00225	.00137	.00033	0
8.0	.00878	.00516	.00339	.00268	.00179	.00111	.00026	0
9.0	.00726	.00423	.00277	.00218	.00146	.00090	.00021	0
10.0	.00611	.00353	.00230	.00182	.00121	.00075	.00018	0

VALUES OF K OBTAINED FROM a = 0.0 COLUMN.

$M_x = \frac{w}{2}(x - al)^2$
$m = x$
$I_x = I(1 + \frac{c}{l}x)^2$



$I_A = I + \frac{btd^2}{2}$ (approx)

$$\Delta_A = \int_{al}^{l} \frac{M_x m \, \delta x}{E I_x} = \frac{wl^4}{EI} \cdot \frac{1}{2c^2} \left\{ (1-a)\left(\frac{1}{2} - \frac{1}{c} - \frac{3a}{2}\right) - \frac{1-a}{1+c}\left(a + 1 + \frac{2}{c}\right) + \left[a\left(a + \frac{4}{c}\right) + \frac{3}{c^2} \right] \log_e \frac{1+c}{1+ca} \right\} = \frac{wl^4}{EI} \cdot k_{w3}$$

VALUES OF k_{w3}

c	a						
	0.0	0.2	0.4	0.5	0.6	0.8	1.0
.0	.12500	.06827	.03060	.01823	.00960	.00127	0
.1	.10724	.05822	.02591	.01538	.00806	.00106	0
.2	.09313	.04931	.02330	.01317	.00688	.00089	0
.3	.08170	.04392	.01932	.01140	.00594	.00077	0
.4	.07231	.03869	.01694	.00996	.00518	.00066	0
.5	.06450	.03436	.01497	.00878	.00455	.00058	0
.6	.05791	.03073	.01333	.00780	.00404	.00051	0
.7	.05231	.02766	.01195	.00698	.00360	.00046	0
.8	.04750	.02502	.01077	.00628	.00324	.00040	0
.9	.04335	.02275	.00976	.00568	.00292	.00037	0
1.0	.03972	.02079	.00889	.00517	.00265	.00032	0
1.2	.03380	.01755	.00746	.00432	.00221	.00028	0
1.4	.02903	.01502	.00635	.00338	.00188	.00023	0
1.6	.02526	.01301	.00547	.00316	.00161	.00020	0
1.8	.02220	.01137	.00477	.00274	.00140	.00017	0
2.0	.01966	.01003	.00418	.00241	.00122	.00015	0
2.25	.01707	.00867	.00360	.00207	.00105	.00013	0
2.5	.01496	.00757	.00313	.00179	.00091	.00011	0
2.75	.01323	.00666	.00275	.00157	.00080	.00010	0
3.0	.01123	.00591	.00243	.00139	.00070	.00009	0
3.5	.00953	.00475	.00194	.00111	.00056	.00007	0
4.0	.00787	.00390	.00159	.00090	.00045	.00006	0
5.0	.00563	.00276	.00112	.00063	.00032	.00004	0
6.0	.00424	.00206	.00083	.00047	.00024	.00003	0
7.0	.00330	.00160	.00064	.00036	.00018	.00002	0
8.0	.00265	.00127	.00051	.00029	.00014	.00002	0
9.0	.00217	.00104	.00041	.00023	.00012	.00001	0
10.0	.00181	.00087	.00034	.00019	.00010	.00001	0

DATE DUE

Thesis Spangler 12848
S6664

A study of the effect of span and number of bays on economy of framing material in single story gabled structures utilizing articulated wedge-beam framing

DATE DUE	BORROWER'S NAME

structures utilizing articulated wedge-bean framing

DUDLEY KNOX LIBRARY
3 2768 00032613 6